Interacting Ethical Fields: The Coupled Dynamics of The Good, The Right, and Virtue

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Abstract

Following John Rawls, we can characterize moral theory as the study of how the basic notions of the good, the right, and moral worth (virtue) may be arranged to form different moral structures. Philosophers working within particular traditions have typically tried to explain which of these notions is more fundamental, attempting to derive or reduce some from or to others. In this paper, I take a different strategy in connecting these different moral notions, one that relates them *dynamically*. I argue that the way these ethical notions relate can be understood as analogous to the way electricity and magnetism relate to each other. Thus, I advance a view on which we can understand axiological phenomena (encompassed by the good), deontic phenomena (encompassed by the right), and aretaic phenomena (encompassed by moral worth) as manifestations of axiological, deontic, and aretaic fields that dynamically couple to each other, analogously to how electric and magnetic fields do. Moreover, I argue that these fields are rank-2 tensor fields, representable as matrices in which the diagonal entries capture the evaluative content along distinct sub-dimensions within each ethical dimension, and the off-diagonal entries capture the synergies and antagonisms between those sub-dimensions. The resulting framework provides a non-reductive unification of the three fundamental ethical dimensions, explains the structural character of genuine ethical dilemmas and moral synergies, illuminates the emergence of non-additive ethical patterns such as virtuous and vicious spirals, and connects to the gauge-theoretic foundations developed in companion work.

Introduction

Rawls offers a characterization of moral theory as "the study of how the basic notions of the right, the good, and moral worth may be arranged to form different moral structures." The suggestion is that the fundamental task of systematic moral philosophy is not simply to defend particular verdicts about particular cases but to articulate how these three basic notions relate to each other: which is prior, which is derivative, how each constrains the others, and what the resulting architecture of normativity looks like.[1]

[1]Rawls (1975, 5). Rawls's own project in *A Theory of Justice* of course, proceeds to develop a particular arrangement in which the right is prior to the good, and in which moral worth is characterized by a desire to

Rawls's characterization points to a fact that is so pervasive in moral philosophy that it can be difficult to see: the three basic ethical dimensions—the axiological (what is good and bad), the deontic (what is right and wrong), and the aretaic (what is virtuous and vicious)—are both irreducible to one another and not independent of one another. Different major traditions in moral philosophy can be understood as particular hypotheses about the priority relations among these three notions: consequentialism takes the good as fundamental and derives the right and virtue from it; deontological theories take the right as fundamental and characterize the good and virtue in terms of their relation to duty; virtue ethics takes character as fundamental and understands the good and the right through the lens of what a virtuous agent would do and care about. Each tradition has generated powerful insights, and each faces well-known difficulties precisely at the points where it attempts to reduce the other dimensions to its own preferred starting point.[2]

The persistence of this tripartite structure across centuries of moral philosophy (and across independent philosophical traditions) suggests that the three dimensions are not easily collapsed. And yet they are manifestly not independent: what is good bears on what is right (consequences are relevant to obligation), what is right bears on what is virtuous (responsiveness to duty is partly constitutive of good character), and what is virtuous bears on what is good (the exercise of virtue contributes to flourishing). The question that Rawls's characterization leaves open is: what is the *structure* of this interdependence?

Most systematic efforts to answer this question have been *reductive*: they have tried to show that one dimension is explanatorily prior to the others, and that the others can be understood as derived from or grounded in the prior dimension. The present paper takes a different approach. Rather than attempting to reduce some of the basic ethical dimensions to others, I propose to *relate them dynamically*. The key insight is drawn from a structural analogy with physics.

Before Maxwell, electricity and magnetism were understood as separate phenomena with observed empirical connections. Hans Christian Ørsted had discovered that electric currents produce magnetic fields; Michael Faraday had discovered that changing magnetic fields produce electric currents. The two phenomena were known to interact, but the nature of their relationship was unclear. Were they fundamentally different kinds of thing that happened to influence each other? Was one more fundamental than the other? Maxwell's great unification showed that these questions were ill-posed. Electric and magnetic fields are not separate entities that happen to interact; they are dynamically coupled aspects of a single electromagnetic field. A changing electric field generates a magnetic field; a changing magnetic field generates an electric field. The two are bound together by coupling equations (Maxwell's equations) that specify how each evolves in response to the other. The unification was *non-*

act from the principles that would be chosen in the original position. The present paper does not presuppose Rawls's particular arrangement. What it takes from Rawls is the characterization of the problem space: the idea that the fundamental notions of moral theory stand in structural relations to one another, and that a central task of moral theory is to articulate those relations.

[2]The difficulties are familiar. Consequentialism struggles to explain the apparent moral significance of distinctions (e.g., between doing and allowing, or between intended and foreseen harm) that resist axiological reduction. Deontological theories face the challenge of explaining why consequences matter without conceding explanatory priority to the good. Virtue ethics must explain how claims about what is virtuous can be non-circular without implicit appeal to an independent standard of the good or the right. These difficulties are not failures of ingenuity; they are symptoms of the underlying irreducibility of the three dimensions.

reductive: Maxwell did not show that magnetism is "really" electricity, or vice versa. He showed that electricity and magnetism are co-fundamental and dynamically intertwined.[3]

I argue that the axiological, deontic, and aretaic dimensions stand in a structurally analogous relationship. Axiological phenomena (encompassed by the good), deontic phenomena (encompassed by the right), and aretaic phenomena (encompassed by moral worth) can be understood as manifestations of three dynamically coupled ethical fields—an axiological field $\mathcal{G}$, a deontic field $\mathcal{R}$, and an aretaic field $\mathcal{V}$—that co-evolve in response to one another. Changes in the landscape of goods and harms ($\mathcal{G}$) generate changes in the structure of obligations and permissions ($\mathcal{R}$); changes in the structure of obligations and permissions generate demands on character and thereby reshape the landscape of virtues and vices ($\mathcal{V}$); and changes in the configuration of character reshape the horizon of achievable goods and the forms of flourishing available to agents and communities ($\mathcal{G}$). The coupling is dynamic and bidirectional, and none of the three fields is explanatorily prior to the others.[4]

Moreover, I argue that each of these fields is a *rank-2 tensor field*, a mathematical object richer than a scalar or a vector, one that captures not only the evaluative content along each sub-dimension within an ethical dimension but also the synergies and antagonisms *between* sub-dimensions. The tensor representation is not an arbitrary formal choice; it is the simplest mathematical structure adequate to the ethical reality. Each of the three fundamental ethical dimensions is itself multidimensional (goodness involves welfare, autonomy, fairness, excellence, and more; rightness involves distinct normative considerations; virtue involves distinct character traits), and the sub-dimensions within each field are coupled to one another in ways that a single number or even a list of numbers cannot capture. The tensor is exactly the mathematical object that encodes the full pattern of interdimensional relationships.

The paper's argument proceeds in five stages. Section 1 develops the philosophical groundwork: the three fundamental dimensions of ethical evaluation, their internal multidimensionality, and the case for their irreducibility and interdependence. Section 2 develops the electromagnetic analogy with sufficient care that a reader without physics background can follow the structural point. Section 3 presents the core formal proposal: the modeling of the three ethical dimensions as dynamically coupled rank-2 tensor fields. Section 4 draws out the philosophical upshots: the non-reductive unification of the three dimensions, the structural characterization of ethical dilemmas and moral synergies, the emergence of non-additive ethical patterns (virtuous and vicious spirals), the illumination of the structure of moral disagreement, and the connection to the gauge-theoretic foundations developed in

[3]I mean the analogy to be structural, not metaphysical. The claim is not that ethical phenomena are electromagnetic phenomena, or that the ethical domain is governed by differential equations of the same type as Maxwell's. The claim is that the *pattern of relationship* between the three basic ethical dimensions—irreducibility combined with dynamic coupling—is structurally analogous to the pattern of relationship between the electric and magnetic fields, and that the mathematical tools developed to represent the latter can be adapted to illuminate the former. The companion papers develop the analogy between physics and ethics at a deeper level, applying the structural logic of symmetry and gauge theory to the foundations of ethical objectivity (Sanchez Borboa, manuscripts A and B). The present paper concerns the "matter" side of the analogy: the structure of the ethical fields themselves, rather than the connection (gauge) field that organizes them.

[4]Throughout the paper, I use $\mathcal{G}$ (the Good) for the axiological field, $\mathcal{R}$ (the Right) for the deontic field, and $\mathcal{V}$ (Virtue) for the aretaic field.

companion work. Section 5 examines the principal disanalogies between the physical and ethical cases and argues that they are philosophically productive rather than problematic. Section 6 concludes.

The argument is, as in the companion papers, conditional and structural. It does not presuppose any particular first-order ethical theory. It shows that *if* we accept that the three basic ethical dimensions are both irreducible and interdependent—a claim that all major traditions have reason to accept—then the dynamic-coupling framework provides a powerful and previously unavailable way of articulating that interdependence, one that generates novel philosophical insights not accessible from within any single tradition.

1 The Three Dimensions of Ethical Evaluation

Ethical evaluation is not monolithic. When we assess a situation, an action, a policy, or a person's character, we draw, often simultaneously, on at least three distinguishable kinds of evaluative consideration. This section characterizes these three dimensions, argues that each is internally multidimensional, and defends the joint claim of irreducibility and interdependence that motivates the dynamic-coupling framework.

1.1 The axiological dimension: the good

The axiological dimension concerns what is good and bad, valuable and disvaluable, better and worse. It is the dimension of evaluation that asks: what states of affairs, experiences, relationships, or conditions are worth pursuing, promoting, or protecting, and which are to be avoided, mitigated, or lamented?

The axiological domain is the natural home of consequentialist moral theory, but it is not the exclusive property of consequentialism. Every ethical tradition recognizes that some things are good and others bad, that welfare matters, that suffering is pro tanto a bad thing, that flourishing is a desirable condition. The axiological dimension is present in the Aristotelian conception of eudaimonia, in the utilitarian calculus of pleasure and pain, in the Kantian recognition that happiness is a legitimate (if conditioned) end, in the Confucian ideal of harmonious social relations, and in the ethics of care's attention to relational well-being. Disagreements arise about what is ultimately good, about whether goodness is agent-neutral or agent-relative, and about the role of the good in the overall structure of morality, but the existence of the axiological dimension as such is common ground.[5]

1.2 The deontic dimension: the right

The deontic dimension concerns what is right and wrong, obligatory and forbidden, permitted and impermissible. It is the dimension of evaluation that asks: what ought one to do? What

[5]I use "axiological" and "the good" as labels for this dimension without presupposing any particular axiology. The framework is compatible with hedonism, preference-satisfaction accounts, objective-list theories, and other substantive theories of value. What matters for the present argument is that the axiological dimension exists and has the structural features described below, not which particular axiology is correct.

are one's duties, obligations, and entitlements? What constraints govern the pursuit of the good?

The deontic domain is the natural home of deontological moral theory, but again, it is not the exclusive property of deontology. Consequentialists acknowledge (or attempt to derive) obligations; virtue ethicists recognize that some actions are right and others wrong, even if they characterize rightness through the dispositions of the virtuous agent. The deontic dimension is present wherever moral philosophy recognizes that there are things one *must* do or *must not* do, irrespective of (or in addition to) considerations of what would produce the best outcome. The structure of the deontic domain is richer than a simple ranking: it involves distinctions between obligations and permissions, between perfect and imperfect duties, between what is owed to specific persons and what is required impartially, between the stringent and the supererogatory.[6]

1.3 The aretaic dimension: virtue/moral worth

The aretaic dimension concerns virtue/moral worth—the virtues and vices of character, the excellences and deficiencies of moral personality. It is the dimension of evaluation that asks: what kind of person is one? What dispositions, sensitivities, and patterns of motivation does one exhibit? What character traits are admirable, and which are contemptible?

The aretaic domain is the natural home of virtue ethics, but it too is not the property of a single tradition. Kantians recognize the moral significance of a good will—indeed, the companion paper argues at length that the good will has the structural profile of a gauge-theoretic connection field.[7] Consequentialists recognize that certain character traits are instrumentally (or even intrinsically) valuable. The aretaic dimension is present wherever moral philosophy attends to the quality of an agent's motivation, not merely to the quality of their actions or the value of their outcomes. The domain has its own internal complexity: virtues differ from one another (courage is not justice, temperance is not practical wisdom), they stand in relationships of mutual support and potential tension, and they admit of degrees.

1.4 Multidimensionality within each dimension

A point of central importance for what follows: each of the three fundamental ethical dimensions is itself *multidimensional*. This point is familiar to practitioners within each tradition, but its formal significance has not been adequately appreciated.

Consider goodness first. A naive model of the axiological dimension might assign a single number, i.e., a scalar degree of goodness, to each ethically evaluable situation. But this

[6] As with the axiological dimension, I use "deontic" and "the right" as labels without presupposing any particular deontological theory. The framework is compatible with Kantian, contractualist, rights-based, and other accounts of the structure of obligation.

[7] Sanchez Borboa (manuscript B). The good will, on the gauge-theoretic reading, is the evaluative structure that enforces coherence across locally varying frames of ethical assessment. The present paper's characterization of the aretaic field $\mathcal{V}$ is compatible with and complementary to that analysis: the gauge connection organizes the ethical matter fields (including $\mathcal{V}$) into frame-independent evaluations, while $\mathcal{V}$ itself encodes the substantive evaluative content concerning character.

is manifestly inadequate for several reasons that ethicists have long recognized. Goodness involves welfare, autonomy, fairness, excellence, relational quality, and other sub-dimensions that are not reducible to a single scale. These sub-dimensions are not merely different *amounts* of a single quantity; they are different *kinds* of evaluative consideration, each with its own internal logic. Welfare admits of a cardinal structure (more or less well-being); autonomy has a threshold character (respected or violated, with degrees of each); fairness is inherently comparative (it concerns distributions across persons); excellence is perfective (it concerns the realization of standards internal to a practice or a form of life). The diversity of these internal logics is precisely what makes scalar aggregation (the summation of all goods into a single "utility" value) philosophically controversial: it requires a common measure where none may exist.[8]

And crucially, these sub-dimensions are not independent. Promoting welfare can enhance or compromise autonomy, depending on context: a paternalistic intervention may increase welfare while diminishing autonomy, whereas empowering someone to make informed choices may serve both. Fairness can synergize with or antagonize relational quality: a strictly impartial distribution may be fair but may also erode the relational bonds that a community depends on, whereas attending to relational quality without regard for fairness can entrench privilege. Excellence can serve or undermine welfare: the demanding pursuit of artistic or athletic excellence can enhance a life or consume it. The ethically significant content is not exhausted by the values along each sub-dimension considered in isolation; it resides equally in the *pattern of relationships among* the sub-dimensions.

Parallel points apply to the deontic dimension. Rightness is not a single thing. The structure of obligation involves distinct normative considerations, e.g., non-maleficence, justice, fidelity, beneficence, respect for autonomy, that can reinforce or conflict with one another. A physician's duty of beneficence may pull in one direction while the duty to respect patient autonomy pulls in another, not because the physician is confused about what "rightness" means but because the deontic dimension has internal structure that generates genuine normative tensions. Different frameworks for organizing these considerations (rights-based, contractualist, role-based) reflect different hypotheses about the internal architecture of the deontic field, not merely different conclusions about which actions are right.

And to the aretaic dimension. The virtues are plural: courage, justice, temperance, generosity, honesty, compassion, practical wisdom. Classical virtue theory has long debated whether the virtues form a unity (the Socratic thesis that virtue is one) or a genuine plurality (the Aristotelian recognition that the virtues are distinct excellences that can, in non-ideal agents, come apart). What is clear is that the aretaic dimension has internal structure: the virtues stand in relationships of mutual support (practical wisdom enhances the exercise of all other virtues), potential tension (courage may conflict with prudence; justice may conflict with mercy), and developmental interdependence (certain virtues are preconditions for the cultivation of others).

[8]The point is related to the classical arguments against simple aggregative consequentialism offered by, among others, Sen (1979) on the informational basis of welfare judgments, Nussbaum (2000) on the irreducible plurality of central capabilities, and Raz (1986) on incommensurability. The tensor framework developed in §3 does not resolve these debates but provides a formal home for the structural complexity they identify.

1.5 Irreducibility and interdependence

The philosophical significance of these observations is this: each of the three fundamental ethical dimensions has a richness of internal structure that resists reduction to either of the other two, and yet the three dimensions are not independent of one another. This combination of irreducibility and interdependence is the central datum that the present paper seeks to model.

1.5.1 Irreducibility

That the three dimensions resist mutual reduction is supported by well-known arguments.

The good cannot be fully captured by the right. Not all goodness is goodness-of-compliance-with-duty: the value of a beautiful sunset, of deep friendship, of the experience of understanding something for the first time, is not exhausted by (and may have nothing to do with) its relation to anyone's obligations. Even if we restrict attention to morally relevant goods, the evaluative richness of the axiological dimension, the fact that it involves welfare, autonomy, excellence, relational quality, and more is not plausibly derivable from the structure of obligation alone.

The right cannot be fully captured by the good. This is the insight that has animated deontological ethics since Kant: the structure of obligation is not exhausted by the optimization of value. The distinction between doing and allowing, between intended and merely foreseen consequences, between what one owes to particular persons and what would produce the best outcome overall, these are features of the deontic landscape that resist axiological reduction, as the companion paper's invariance arguments help to explain.[9]

Moral worth cannot be fully captured by either the good or the right. The quality of an agent's character, i.e., their dispositions, sensitivities, and patterns of motivation, is not fully determined by the outcomes they produce (someone can produce good outcomes from bad motives, or bad outcomes from good motives) or by their compliance with duty (someone can comply with duty grudgingly, mechanically, or from fear, in ways that are deontically correct but aretaically deficient). This is, in essence, the Kantian point that moral worth attaches not to the action's conformity with duty but to the agent's acting *from* duty—a point that the companion paper gives gauge-theoretic expression.

1.5.2 Interdependence

And yet the three dimensions are not independent. Their interdependence is manifest in the way considerations from each dimension bear on the others.

Changes in the axiological landscape generate deontic consequences: the discovery that a widely used chemical causes severe environmental harm generates new obligations (e.g., to cease its use, to remediate damage, to compensate those affected) that did not exist (or

[9]The companion paper (Sanchez Borboa, manuscript A) argues that the structural priority of moral reasons over merely prudential considerations follows from the symmetry requirements constitutive of practical objectivity. This result supports the irreducibility of the deontic: if the right had its authority solely in virtue of its promotion of the good, then moral reasons would be a species of prudential reasons (reasons to maximize value), and their structural priority would be unexplained.

were not recognizable) before the axiological facts became clear. Changes in what is good (or bad) reshape what is right.

Changes in the deontic landscape generate aretaic consequences: when new duties are recognized (for example, duties of environmental stewardship, or duties arising from the recognition of previously invisible forms of injustice) the character traits that constitute virtue shift accordingly. A person who was considered virtuous a century ago for traits that are now recognized as complicit with injustice is re-evaluated in light of the changed deontic landscape. Changes in what is right reshape what is virtuous.[10]

Changes in the aretaic landscape generate axiological consequences: when a community cultivates new virtues (for instance, virtues of ecological attentiveness, or virtues of cross-cultural empathy) the forms of flourishing available to its members expand. New goods become accessible, new forms of relational quality become possible, new dimensions of excellence open up. Changes in what is virtuous reshape what is good.

These observations are not novel. Philosophers across traditions have recognized each of these connections individually. What has been lacking is a formal framework that represents the connections *simultaneously* and *structurally*, one that captures not only the existence of interdependence but its *pattern*: which dimensions couple to which, how strongly, in which direction, and with what emergent consequences. That is the task of the remainder of this paper. The next section develops the physical analogy that motivates the formal apparatus; Section 3 presents the apparatus itself.

2 The Electromagnetic Analogy

The previous section established that the three fundamental dimensions of ethical evaluation (the axiological, the deontic, and the aretaic) are both internally multidimensional and mutually interdependent in ways that resist reduction. The question is what formal structure can represent this combination of irreducibility and interdependence. This section develops an analogy with the history of electromagnetism that motivates the formal apparatus of §3. The analogy is structural, not metaphysical: it identifies a pattern of relationship (irreducible-but-dynamically-coupled fields representable by a tensor) that has proven enormously fruitful in physics and that, I shall argue, illuminates the ethical case with genuine precision.[11]

[10] The point is not that moral standards are merely conventional or historically contingent. It is that the substantive content of virtue is responsive to the structure of obligation, which is itself responsive to (among other things) the axiological landscape. A genuine expansion of moral knowledge—recognizing a form of harm or injustice that was previously invisible—generates legitimate pressure on the content of the other two dimensions. The dynamic coupling is a coupling between genuine ethical content, not between mere social conventions.

[11] A reader who is already persuaded that the three ethical dimensions are dynamically coupled and who is comfortable with rank-2 tensors may wish to skim this section and proceed directly to §3. The section is written for readers who want to see why the tensor-field framework is motivated by more than formal convenience.

2.1 Electricity and magnetism before Maxwell

For most of the history of physics, electricity and magnetism were understood as separate phenomena. Electric charges exert forces on one another; magnets attract and repel. The two sets of phenomena were studied by different investigators, described by different laws, and understood through different conceptual frameworks.

The first cracks in this separation appeared empirically. In 1820, Hans Christian Ørsted discovered that an electric current (a flow of charges) produces a magnetic field: a compass needle placed near a current-carrying wire deflects. The effect was directional and systematic, not a loose correlation. Shortly afterward, André-Marie Ampère showed that two current-carrying wires exert magnetic forces on each other, and he formulated quantitative laws governing these forces. In the 1830s, Michael Faraday discovered the converse effect: a *changing* magnetic field produces an electric current in a nearby conductor. This is electromagnetic induction, the principle behind electrical generators and transformers.

These discoveries established that electricity and magnetism are connected. But the nature of the connection remained unclear. Was electricity more fundamental, with magnetism a secondary effect of moving charges? Was magnetism more fundamental? Were they two independent phenomena that happened to influence each other through empirical laws? The situation, in retrospect, was structurally analogous to the situation in moral philosophy described in §1: two (in the ethical case, three) apparently distinct domains, known to be interdependent, with the *structure* of their interdependence unresolved.

2.2 Maxwell's unification

James Clerk Maxwell resolved the question in the 1860s by showing that electricity and magnetism are not separate phenomena that happen to interact but dynamically coupled aspects of a single electromagnetic field. The key structural insight can be stated without equations:[12]

- A changing electric field generates a magnetic field. (This was Maxwell's own novel contribution, the "displacement current" that completed the system.)
- A changing magnetic field generates an electric field. (This is Faraday's law of induction.)

The two fields are bound together by *coupling equations*: Maxwell's equations, which that specify precisely how each field evolves in response to the other's configuration and rate of change. The coupling is *dynamic*: it is not that one field is defined in terms of the other (that would be reduction), nor that they merely happen to co-occur (that would be coincidence). Rather, the evolution of each field is constitutively governed by the configuration of the other.

[12]The non-mathematical reader loses nothing essential by taking the structural points on trust and skipping the mathematical details. The formal content of the analogy for the ethical case is developed independently in §3.

The electric field at a point depends on what the magnetic field is doing in the neighborhood of that point, and vice versa.[13]

Three features of Maxwell's unification are especially relevant for the ethical analogy.

First, the unification is *non-reductive*. Maxwell did not show that magnetism is "really" electricity, or that electric phenomena can be derived from magnetic ones. He showed that the two are co-fundamental: neither is explanatorily prior; both are necessary components of a single field whose dynamics are governed by their mutual coupling. The electric field without the magnetic field is not a self-contained physical story, and the magnetic field without the electric field is not either. They are, individually, abstractions from a richer whole.

Second, the coupling generates *emergent phenomena* that belong to neither field in isolation. Electromagnetic waves (light) are the paradigmatic example. A changing electric field generates a changing magnetic field, which generates a changing electric field, and so on: the mutual coupling sustains a self-propagating oscillation that travels through space. Light is not an electric phenomenon or a magnetic phenomenon; it is an electromagnetic phenomenon, an emergent product of the dynamic coupling between the two fields. Without the coupling, light does not exist.

Third, the unification is *field-theoretic*: the electric and magnetic fields are not properties of individual charged particles but structures defined at every point of space, varying continuously from place to place and moment to moment. The field concept is what allows the coupling to be *local*, i.e., the electric field at a point couples to the magnetic field in its immediate neighborhood, not to distant magnetic configurations, and it is what makes the emergent phenomena (waves, radiation) possible.[14]

2.3 The electromagnetic field tensor

Maxwell's equations, as originally formulated, are a system of coupled partial differential equations relating the electric and magnetic fields (each a three-component vector in space). A deeper mathematical representation, developed in the context of special relativity, unifies the two fields into a single mathematical object: the *electromagnetic field tensor* $F_{\mu\nu}$, also called the Faraday tensor.

The Faraday tensor is a rank-2 tensor: a mathematical object naturally represented as a 4×4 matrix (in the four-dimensional spacetime of special relativity) whose components encode both the electric and magnetic fields. In a suitable decomposition:[15]

[13] More precisely: in source-free regions, the time derivative of the electric field is proportional to the curl of the magnetic field, and the time derivative of the magnetic field is proportional to the (negative) curl of the electric field. These are two of Maxwell's four equations. The other two (Gauss's laws for electricity and magnetism) constrain the fields' spatial structure. For accessible treatments, see, e.g., Griffiths (2017) or Purcell and Morin (2013).

[14] The field concept was Faraday's great conceptual contribution, one that Maxwell formalized mathematically. The transition from "forces between particles" to "fields pervading space" is one of the most consequential shifts in the history of physics. I discuss the ethical significance of the field concept—as opposed to evaluations attached to individual cases in isolation—in §3.5.

[15] Specifically, in components relative to a given inertial frame, the six independent components of the antisymmetric 4×4 Faraday tensor decompose into three electric-field components (F_{0i} for $i = 1, 2, 3$) and three magnetic-field components (F_{ij} for $i, j = 1, 2, 3$, $i \neq j$). The decomposition is frame-dependent—

- Certain components of the tensor encode the electric field.
- Other components encode the magnetic field.
- The tensor as a whole encodes the complete electromagnetic field, including the coupling between its electric and magnetic aspects.

What makes the tensor representation significant, for our purposes, is that it captures two features simultaneously: the *distinctness* of the electric and magnetic fields (they correspond to different components) and their *structural unity* (they are components of a single geometric object whose transformation properties are well defined). The tensor does not merely list the electric and magnetic field values side by side; it represents them as aspects of a unified structure that transforms as a whole when one changes coordinates (reference frame).

A key mathematical property: the Faraday tensor is *antisymmetric*: $F_{\mu\nu} = -F_{\nu\mu}$. This means that the coupling between the electric and magnetic components has a specific mathematical character forced by the structure of spacetime and the Lorentz group. As we shall see in §3, the ethical tensor will be *general*, i.e., neither symmetric nor antisymmetric, reflecting a richer coupling structure in the ethical domain. This is a productive disanalogy rather than a defect, and I develop its significance in §§3.4 and 5.5.

2.4 What the analogy suggests

The electromagnetic analogy suggests a specific structural hypothesis about the relationship among the three fundamental ethical dimensions:

> The axiological ($\mathcal{G}$), deontic ($\mathcal{R}$), and aretaic ($\mathcal{V}$) dimensions are dynamically coupled aspects of a unified ethical field structure, representable by rank-2 tensor fields whose components encode both the evaluative content along each sub-dimension and the coupling between sub-dimensions.

The analogy is not identity. The ethical case differs from the electromagnetic case in several important respects that I examine in §5. But the structural parallels are genuine and, I shall argue, generative:

- Where Maxwell found two fields (electric and magnetic) that are irreducible to each other yet dynamically coupled, the ethical case presents three fields ($\mathcal{G}$, $\mathcal{R}$, $\mathcal{V}$) with the same pattern of irreducibility and interdependence.
- Where Maxwell showed that the coupling generates emergent phenomena (electromagnetic waves) that belong to neither field alone, the ethical coupling generates emergent moral patterns (e.g., virtuous and vicious spirals, moral synergies, and systemic ethical dynamics) that no single dimension can account for (§4).

what one observer calls a pure electric field, another observer in relative motion may decompose into a mixture of electric and magnetic components—but the tensor itself is a frame-independent geometric object. This is precisely the kind of frame-independence that the companion papers' invariance framework demands of objective ethical evaluation.

- Where the Faraday tensor captures both the distinctness and the unity of the electric and magnetic fields in a single mathematical object, the ethical tensor captures both the distinctness and the interdependence of the ethical sub-dimensions within each fundamental field.
- Where the field concept allows the electromagnetic coupling to be local (each point in space has its own field values), the field concept in the ethical case allows the coupling to be situation-dependent: the pattern of interdependence between the ethical dimensions varies across the ethical landscape, from case to case, in ways that a fixed global relationship cannot capture.

The next section develops the formal content of this hypothesis.

3 Ethical Fields as Rank-2 Tensor Fields

This section presents the core formal proposal. It argues that each of the three fundamental ethical dimensions—the axiological ($\mathcal{G}$), the deontic ($\mathcal{R}$), and the aretaic ($\mathcal{V}$)—is appropriately modeled as a rank-2 tensor field over its internal sub-dimensions, and that the three fields are dynamically coupled to one another. The argument proceeds from the inadequacy of simpler mathematical representations (§3.1), through the intra-dimensional tensor structure (§3.2), to the inter-dimensional coupling (§3.3), the case for general (asymmetric) tensors (§3.4), the field character of the construction (§3.5), the structure of the dynamic coupling (§3.6), and a formal summary (§3.7).

3.1 Why not scalars or vectors?

The simplest conceivable formal model of an ethical dimension would assign a single number—a scalar—to each ethically evaluable situation: a "degree of goodness," a "degree of rightness," or a "degree of virtue." Such scalar models are implicit in many philosophical discussions (the utilitarian's "utility function" is a scalar model of goodness) and explicit in much formal ethics and social choice theory.

The argument of §1.4 has already shown why scalar models are inadequate: each ethical dimension is internally multidimensional, and the sub-dimensions have different internal logics (cardinal, threshold, comparative, perfective) that resist aggregation into a single number. But it is worth being precise about the formal point, because it motivates the move to tensors rather than merely to vectors.

A *vector* model would assign to each situation a list of numbers (one for each sub-dimension) rather than a single number. A vector model of goodness, for instance, might assign a welfare value, an autonomy value, a fairness value, and an excellence value to each situation. This is a genuine improvement over the scalar model: it preserves the multidimensionality of the evaluation and avoids the controversial aggregation step.

But a vector model, for all its advantages over a scalar model, is still inadequate to the ethical reality. A vector records the *magnitudes* along each sub-dimension, but it cannot represent the *relationships between* sub-dimensions. It cannot capture the fact that in a given situation, welfare and autonomy are synergistic (promoting one promotes the other),

while fairness and relational quality are antagonistic (promoting one comes at the cost of the other). It cannot represent the fact that these relationships vary from situation to situation: in one context, welfare and autonomy align; in another, they pull apart.

A rank-2 tensor is exactly the mathematical object that captures this richer structure. If the ethical dimension in question has n sub-dimensions, the corresponding rank-2 tensor is an $n \times n$ matrix $\mathbf{T}$ whose entries encode both the self-contributions of each sub-dimension (the diagonal entries T_{ii}) and the cross-dimensional couplings (the off-diagonal entries T_{ij}, $i \neq j$). The diagonal entry T_{ii} records how much the ith sub-dimension contributes to the overall evaluative profile; the off-diagonal entry T_{ij} records how the ith and jth sub-dimensions bear on each other in the situation at hand.[16]

The progression from scalar to vector to tensor is not a matter of increasing mathematical sophistication for its own sake. It is driven by the structure of the subject matter: the ethical reality has coupling structure that simpler mathematical objects cannot represent. The tensor is the simplest mathematical object adequate to the task.

3.2 The intra-dimensional tensor: multidimensional goodness, rightness, and virtue

I now apply this general point to each of the three fundamental ethical dimensions.

3.2.1 The axiological tensor $\mathcal{G}$

Let the axiological dimension have $n_{\mathcal{G}}$ sub-dimensions (e.g., welfare (g_1), autonomy (g_2), fairness (g_3), excellence (g_4), relational quality (g_5), and potentially others). The axiological tensor $\mathcal{G}$ is an $n_{\mathcal{G}} \times n_{\mathcal{G}}$ matrix whose entries encode the evaluative content of goodness in a given ethical situation:

$$\mathcal{G} = \begin{bmatrix} \mathcal{G}_{11} & \mathcal{G}_{12} & \mathcal{G}_{13} & \cdots \\ \mathcal{G}_{21} & \mathcal{G}_{22} & \mathcal{G}_{23} & \cdots \\ \mathcal{G}_{31} & \mathcal{G}_{32} & \mathcal{G}_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

The diagonal entries $\mathcal{G}_{ii}$ represent the self-contribution of each axiological sub-dimension: how much welfare is at stake ($\mathcal{G}_{11}$), how much autonomy is at stake ($\mathcal{G}_{22}$), and so on. These are the quantities that a vector model would capture.

The off-diagonal entries $\mathcal{G}_{ij}$ (where $i \neq j$) represent the cross-dimensional couplings within the axiological field: the synergies and antagonisms between sub-dimensions of goodness. A positive value of $\mathcal{G}_{12}$ indicates that welfare and autonomy are synergistic in this situation (promoting one promotes the other). A negative value indicates antagonism (promoting one comes at the cost of the other). The magnitude indicates the strength of the coupling.

[16] A rank-2 tensor over an n-dimensional space has n^2 independent components (in the general case). A vector over the same space has only n components. The additional $n^2 - n = n(n-1)$ components are precisely the off-diagonal coupling terms—the information about inter-dimensional relationships that the vector model discards. As n grows, the coupling terms become an increasingly dominant fraction of the total information: for $n = 5$, there are 20 coupling terms and 5 diagonal terms. The ethical reality, as argued in §1.4, lives at least as much in the couplings as in the individual dimensions.

Consider a concrete example. A policy of providing unconditional basic income might, in a given context, have the following axiological profile: high welfare contribution ($\mathcal{G}_{11}$ large and positive), moderate autonomy contribution ($\mathcal{G}_{22}$ positive), strong synergy between welfare and autonomy ($\mathcal{G}_{12}$ positive: the policy promotes welfare *by enhancing* autonomy), mild antagonism between welfare and excellence ($\mathcal{G}_{14}$ slightly negative: universal provision may, in some analyses, reduce incentives for certain forms of excellence), and moderate synergy between autonomy and relational quality ($\mathcal{G}_{25}$ positive: autonomous agents can form richer relationships). The full axiological tensor captures this complex evaluative profile in a way that no scalar or vector can.

3.2.2 The deontic tensor $\mathcal{R}$

Let the deontic dimension have $n_{\mathcal{R}}$ sub-dimensions (for instance, non-maleficence (r_1), justice (r_2), fidelity (r_3), beneficence (r_4), respect for autonomy (r_5)). The deontic tensor $\mathcal{R}$ is an $n_{\mathcal{R}} \times n_{\mathcal{R}}$ matrix with the same structure:

$$\mathcal{R} = \begin{bmatrix} \mathcal{R}_{11} & \mathcal{R}_{12} & \mathcal{R}_{13} & \cdots \\ \mathcal{R}_{21} & \mathcal{R}_{22} & \mathcal{R}_{23} & \cdots \\ \mathcal{R}_{31} & \mathcal{R}_{32} & \mathcal{R}_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

The diagonal entries represent the normative demands along each deontic sub-dimension: how strongly non-maleficence applies ($\mathcal{R}_{11}$), how pressing the demands of justice are ($\mathcal{R}_{22}$), and so on. The off-diagonal entries represent the couplings between deontic considerations. A physician facing a treatment decision might encounter positive coupling between non-maleficence and beneficence ($\mathcal{R}_{14} > 0$: avoiding harm and promoting well-being pull in the same direction) but negative coupling between beneficence and respect for autonomy ($\mathcal{R}_{45} < 0$: the best medical outcome conflicts with the patient's stated preferences). Again, these couplings are situation-dependent: in another case, beneficence and respect for autonomy may align perfectly.

3.2.3 The aretaic tensor $\mathcal{V}$

Let the aretaic dimension have $n_{\mathcal{V}}$ sub-dimensions (courage (v_1), justice (v_2), temperance (v_3), practical wisdom (v_4), generosity (v_5), honesty (v_6), and others). The aretaic tensor $\mathcal{V}$ is an $n_{\mathcal{V}} \times n_{\mathcal{V}}$ matrix:

$$\mathcal{V} = \begin{bmatrix} \mathcal{V}_{11} & \mathcal{V}_{12} & \mathcal{V}_{13} & \cdots \\ \mathcal{V}_{21} & \mathcal{V}_{22} & \mathcal{V}_{23} & \cdots \\ \mathcal{V}_{31} & \mathcal{V}_{32} & \mathcal{V}_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

The diagonal entries represent how much each virtue (or vice) is at stake or in play. The off-diagonal entries represent the coupling between virtues. The classical thesis of the unity of the virtues corresponds, in this framework, to the claim that the off-diagonal entries of $\mathcal{V}$ are generally positive: the virtues reinforce one another. An agent cannot be truly

courageous without practical wisdom (because courage without wisdom is recklessness), and cannot be truly just without temperance (because justice without temperance is rigidity). This thesis has been contested; the tensor framework does not presuppose it. It provides a formal home for both the unity thesis ($\mathcal{V}_{ij} \geq 0$ generally) and its denial ($\mathcal{V}_{ij} < 0$ for some virtue-pairs in some situations), making the disagreement structurally precise rather than merely verbal.

3.3 The inter-dimensional coupling: how the three fields interact

The intra-dimensional tensors $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ characterize the internal structure of each ethical dimension. But the argument of §1.5 established that the three dimensions are also coupled to *one another*. This inter-dimensional coupling is a second, higher-order structural feature that the framework must represent.

The inter-dimensional coupling operates at the level of the tensor fields themselves: changes in the axiological tensor $\mathcal{G}$ drive changes in the deontic tensor $\mathcal{R}$ and the aretaic tensor $\mathcal{V}$, and vice versa. This coupling is not merely the claim that the three dimensions are "related", a claim so weak as to be nearly vacuous. It is the claim that the *internal structure* of each dimension is partly determined by the configuration of the others. The coupling is constitutive, not merely causal.

Consider how this works concretely.

$\mathcal{G} \to \mathcal{R}$ coupling (axiological to deontic). When the axiological landscape shifts, when new goods become available, new harms are discovered, or the pattern of synergies and antagonisms among goods changes, the deontic landscape shifts in response. The discovery that a widely used industrial process causes severe environmental harm does not merely add a negative entry to the diagonal of $\mathcal{G}$ (a new harm); it restructures $\mathcal{R}$ by generating new obligations (cessation, remediation, compensation) and altering the coupling between existing deontic sub-dimensions (the relationship between industrial productivity and non-maleficence shifts from benign to antagonistic). The change in $\mathcal{G}$ propagates through to $\mathcal{R}$.

$\mathcal{R} \to \mathcal{V}$ coupling (deontic to aretaic). When the structure of obligations changes, the character traits that constitute virtue change accordingly. The recognition of duties of environmental stewardship generates new demands on character: virtues of ecological attentiveness, restraint in resource use, and sensitivity to non-human welfare become part of the aretaic landscape where previously they were not (or were marginal). The internal coupling structure of $\mathcal{V}$ itself shifts: practical wisdom now requires ecological awareness in a way it previously did not, creating a new positive coupling between the practical-wisdom and ecological-attentiveness components of $\mathcal{V}$ that is driven by the change in $\mathcal{R}$.

$\mathcal{V} \to \mathcal{G}$ coupling (aretaic to axiological). When a community cultivates new virtues, the axiological landscape expands. Virtues of cross-cultural empathy, for instance, make new forms of relational quality and mutual understanding accessible, forms of goodness that were not available (or not recognizable) before those virtues were cultivated. The diagonal of $\mathcal{G}$ acquires new positive entries, and the off-diagonal coupling structure of $\mathcal{G}$ changes as well: the synergy between relational quality and welfare increases when agents are disposed to attend to one another's perspectives.

These three coupling directions do not exhaust the inter-dimensional dynamics. There

are also $\mathcal{R} \to \mathcal{G}$ couplings (the structure of rights and obligations shapes which goods are achievable and how they relate), $\mathcal{V} \to \mathcal{R}$ couplings (the character traits prevalent in a community shape which obligations are recognizable and how they are weighted), and $\mathcal{G} \to \mathcal{V}$ couplings (the landscape of goods and harms shapes which character traits count as virtues and how the virtues relate to one another). Each of the six coupling directions has its own character, and none is reducible to any other.

3.4 General tensors and asymmetric coupling

A point of formal and philosophical importance: the ethical tensors $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ should be *general* rank-2 tensors, that is, they should not be assumed to be either symmetric ($T_{ij} = T_{ji}$) or antisymmetric ($T_{ij} = -T_{ji}$).

This is philosophically motivated. The coupling between any two sub-dimensions is *directional*: the effect of the ith sub-dimension on the jth need not be the same as the effect of the jth on the ith. Consider, within the axiological dimension, the coupling between welfare (g_1) and autonomy (g_2). The way welfare considerations bear on autonomy is structurally different from the way autonomy considerations bear on welfare. Promoting someone's welfare can undermine their autonomy (paternalism), but promoting their autonomy can undermine their welfare in a quite different way (by empowering self-destructive choices). The asymmetry is not a matter of sign (both couplings can be negative) but of *kind*: the mechanism, the moral significance, and the appropriate response differ depending on the direction of influence. A symmetric tensor would force $\mathcal{G}_{12} = \mathcal{G}_{21}$, which would be a substantive (and implausible) ethical assumption smuggled into the formalism: the claim that the coupling between any two sub-dimensions is always bidirectionally identical.[17]

This is a productive disanalogy with the electromagnetic case. The Faraday tensor $F_{\mu\nu}$ is antisymmetric ($F_{\mu\nu} = -F_{\nu\mu}$), and this antisymmetry is not an accidental feature but a consequence of the mathematical structure of spacetime and the Lorentz group: the tensor must be antisymmetric to be consistent with the way electric and magnetic fields transform under changes of reference frame. In the ethical case, no analogous symmetry group constrains the tensor to be symmetric, antisymmetric, or to have any particular index symmetry. The ethical tensor is *general*: its symmetry properties are an open empirical-normative question about the structure of ethical reality, not a formal constraint imposed a priori.[18]

The generality of the ethical tensor is a strength, not a weakness: it means the formalism does not prejudge the structure of ethical coupling but provides a mathematical home in

[17] The point applies at the inter-dimensional level as well. The way the axiological field $\mathcal{G}$ drives changes in the deontic field $\mathcal{R}$ is structurally different from the way $\mathcal{R}$ drives changes in $\mathcal{G}$. The former involves the generation of new obligations in response to shifts in the landscape of goods and harms; the latter involves the shaping of what counts as good by the structure of rights and duties. These are different kinds of moral process, and a formal framework that forced them to be symmetric would be distorting the ethical reality.

[18] This does not mean that the ethical tensor has no structure at all. The companion papers' invariance requirements (§3.7) constrain the tensor's transformation properties: it must transform covariantly under changes of evaluative frame, so that its geometric content (as opposed to its component representation) is frame-independent. What is left open is the *index symmetry* of the tensor—the relationship between T_{ij} and T_{ji}—which is a substantive question about the directionality of ethical coupling rather than a formal constraint of the invariance framework.

which the full range of possible coupling structures (symmetric, antisymmetric, or mixed) can be represented and investigated. Different ethical theories, or different analyses of particular moral situations, may yield tensors with different symmetry properties. The framework accommodates them all.

3.5 The field character of the tensor

I have been speaking of "tensor fields" rather than mere "tensors," and the distinction matters. A tensor is a mathematical object defined at a single point; a tensor *field* is a tensor-valued function that assigns a tensor to each point in a domain. In the ethical case, the relevant domain is the space W of ethically relevant cases introduced in the companion papers.[19]

The ethical field $\mathcal{G}$ is not a single fixed matrix that characterizes goodness once and for all. It is a function $\mathcal{G} : W \to \mathbb{R}^{n_\mathcal{G} \times n_\mathcal{G}}$ that assigns an axiological tensor to each ethically evaluable situation $w \in W$. The axiological coupling between welfare and autonomy may be strongly synergistic in one situation and antagonistic in another; the tensor field captures this variation. Similarly, $\mathcal{R} : W \to \mathbb{R}^{n_\mathcal{R} \times n_\mathcal{R}}$ and $\mathcal{V} : W \to \mathbb{R}^{n_\mathcal{V} \times n_\mathcal{V}}$ are field-valued functions over the same domain.

The field concept is not a gratuitous importation from physics. It is demanded by the ethical reality. The pattern of evaluative content, i.e., which sub-dimensions are salient, how they couple, which synergies and antagonisms obtain, varies from situation to situation. A fixed tensor could not represent this variation; a field can. Moreover, the field concept enables the key structural claim of the paper: that the three fields $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ are *dynamically coupled*, meaning that the configuration of each field at a point in the ethical landscape partially determines the configuration of the others. Without the field concept, "dynamic coupling" would be a metaphor; with it, the coupling can be represented precisely, as a set of structural relationships between the values of the three fields across the ethical landscape.

There is a further philosophical payoff of the field concept. Much of ethical theory proceeds by evaluating individual cases in isolation: *this* action is right, *that* outcome is good, *this* character trait is virtuous. The field concept embeds individual evaluations in a larger landscape, one in which each case is situated among its neighbors and in which the evaluative content at one point is structurally related to the evaluative content at nearby points. This enables the framework to represent not only static evaluations but evaluative gradients, patterns of variation, and (most importantly for the dynamic coupling) the way shifts in one part of the ethical landscape propagate to others. The spiraling dynamics discussed in §4 are field-level phenomena: they depend on the spatial (or, more precisely, the case-topological) structure of the ethical landscape, not merely on the evaluation of individual situations.

3.6 Dynamic coupling: structural constraints on co-evolution

The inter-dimensional coupling described in §3.3 is not unconstrained. Not just any pattern of influence among $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ is consistent with what we know about the ethical domain.

[19] See Sanchez Borboa (manuscript A, appendix) and (manuscript B, §2.1) for the formal characterization of W and its role in the invariance framework.

The present section identifies structural constraints on the coupling, i.e., the "ethical coupling principles" that function as the ethical analogs of Maxwell's equations.[20]

I identify three coupling principles, one for each pair of fundamental ethical dimensions.

Axiological-Deontic Coupling. Changes in the distribution of goods and harms generate deontic structure. When new forms of good or harm become salient (a new technology creates novel risks; a previously invisible form of suffering is recognized), the landscape of obligations adjusts: new duties arise, existing duties are re-weighted, and the coupling structure *within* $\mathcal{R}$ shifts in response to the new configuration of $\mathcal{G}$. Conversely, the structure of obligations constrains and shapes what counts as genuinely good. Not everything that satisfies a desire is good if obtaining it requires the violation of duty; the deontic structure of the situation conditions the axiological status of outcomes. The coupling is bidirectional but asymmetric: the mechanism by which $\mathcal{G}$ generates deontic consequences is not the same as the mechanism by which $\mathcal{R}$ conditions axiological content.

Deontic-Aretaic Coupling. The structure of obligations generates demands on character. Duties that are reliably demanding call forth the cultivation of corresponding virtues: a community with stringent duties of mutual aid develops virtues of generosity and solidarity; a community with complex duties of justice develops virtues of discernment and impartiality. The coupling runs from $\mathcal{R}$ to $\mathcal{V}$. In the other direction, the configuration of virtue shapes which duties are recognizable and how they are met. An agent with well-developed practical wisdom recognizes obligations that a less perceptive agent would miss; a community with cultivated virtues of compassion responds to the demands of beneficence with a sensitivity and precision that a less virtuous community cannot achieve. The coupling from $\mathcal{V}$ to $\mathcal{R}$ is not merely a claim about the *execution* of duty but about its *content*: the duties that are recognizable from a virtuous standpoint are richer than those recognizable from a less developed one.[21]

Aretaic-Axiological Coupling. The cultivation of virtues expands the landscape of achievable goods. New forms of flourishing, new dimensions of relational quality, new modes of excellence become accessible when agents develop the character traits that make them possible. The coupling from $\mathcal{V}$ to $\mathcal{G}$ is a coupling between what agents *are* (in terms of character) and what is *good* (in terms of value). In the other direction, the structure of genuine goods calls forth the development of corresponding virtues: a world in which deep friendship is a central good is a world that calls for virtues of loyalty, vulnerability, and attentiveness; a world in which intellectual understanding is a central good calls for virtues of curiosity, honesty, and persistence. The axiological landscape generates aretaic demands.

These coupling principles are formulated at the structural level and are compatible with different first-order ethical theories. Consequentialists, deontologists, and virtue ethicists

[20] I do not claim that the ethical coupling principles have the quantitative precision of Maxwell's equations. The analogy is structural: just as Maxwell's equations specify how the electric and magnetic fields co-evolve, the ethical coupling principles specify, at the structural level, how the axiological, deontic, and aretaic fields co-evolve. The question of whether more quantitatively precise coupling principles can be formulated for the ethical case is an open question for future work; what the present paper provides is the structural framework within which such a question can be posed.

[21] This point has an affinity with the Aristotelian idea that the phronimos (the person of practical wisdom) perceives moral features of situations that others miss. In the tensor-field framework, this amounts to the claim that the configuration of $\mathcal{V}$ partially determines which components of $\mathcal{R}$ are "activated" in a given situation.

will all recognize these interdependencies, even if they disagree about which dimension has explanatory priority or about the relative strength of the couplings. The framework does not adjudicate that disagreement; it provides a common formal language in which the disagreement can be stated precisely.

3.7 Formal summary

I collect the formal elements of the framework for reference.

Definition 1 (**Ethical Field Domain**). *Let W be a domain of ethically relevant cases, as in the companion papers. Each case $w \in W$ is a complete specification of an ethically evaluable situation (an action, maxim, policy, social arrangement, etc.).*

Definition 2 (**Ethical Dimension Fields**). *The three fundamental ethical dimension fields are tensor-valued functions over W:*

$$\mathcal{G} : W \to \mathbb{R}^{n_\mathcal{G} \times n_\mathcal{G}} \qquad \textit{(the axiological field)} \tag{1}$$

$$\mathcal{R} : W \to \mathbb{R}^{n_\mathcal{R} \times n_\mathcal{R}} \qquad \textit{(the deontic field)} \tag{2}$$

$$\mathcal{V} : W \to \mathbb{R}^{n_\mathcal{V} \times n_\mathcal{V}} \qquad \textit{(the aretaic field)} \tag{3}$$

where $n_\mathcal{G}$, $n_\mathcal{R}$, and $n_\mathcal{V}$ are the number of sub-dimensions within each fundamental dimension. Each field assigns a general (not necessarily symmetric or antisymmetric) rank-2 tensor to each case $w \in W$.

Definition 3 (**Component Interpretation**). *For any ethical dimension field $\mathcal{F} \in \{\mathcal{G}, \mathcal{R}, \mathcal{V}\}$ and case $w \in W$:*

- *The diagonal entry $\mathcal{F}_{ii}(w)$ represents the evaluative content along the ith sub-dimension of $\mathcal{F}$ in case w.*
- *The off-diagonal entry $\mathcal{F}_{ij}(w)$ $(i \neq j)$ represents the coupling from the ith sub-dimension to the jth sub-dimension in case w: positive values indicate synergy; negative values indicate antagonism; the magnitude indicates the strength of the coupling.*
- *The asymmetry $\mathcal{F}_{ij}(w) - \mathcal{F}_{ji}(w)$ represents the directional asymmetry of the coupling: the extent to which the i-to-j influence differs from the j-to-i influence.*

Definition 4 (**Dynamic Inter-field Coupling**). *The three fields are dynamically coupled: the configuration of each field at a case $w \in W$ partially determines the configuration of the others. Schematically:*[22]

$$\Delta\mathcal{R}(w) \sim C^{\mathcal{G}\to\mathcal{R}}\big[\mathcal{G}(w)\big] \cdot C^{\mathcal{V}\to\mathcal{R}}\big[\mathcal{V}(w)\big] \tag{4}$$

$$\Delta\mathcal{G}(w) \sim C^{\mathcal{R}\to\mathcal{G}}\big[\mathcal{R}(w)\big] \cdot C^{\mathcal{V}\to\mathcal{G}}\big[\mathcal{V}(w)\big] \tag{5}$$

$$\Delta\mathcal{V}(w) \sim C^{\mathcal{G}\to\mathcal{V}}\big[\mathcal{G}(w)\big] \cdot C^{\mathcal{R}\to\mathcal{V}}\big[\mathcal{R}(w)\big] \tag{6}$$

[22] A more specific conjectural form of the inter-field coupling — proposed structural field equations governing the co-evolution of $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$, including the multiplicative coupling structure, damping and driving terms reflecting the agent-dependent character of the ethical fields, and the resulting ethical wave equation — is developed in the Appendix. The precise form of these equations is a task for future work; what the Appendix provides is a formal specification of the coupling architecture that makes the dynamical content of the present section's structural principles mathematically explicit.

where $\Delta\mathcal{F}(w)$ denotes the shift in field $\mathcal{F}$ at case w, $C^{\mathcal{A}\to\mathcal{B}}$ denotes the coupling operator from field $\mathcal{A}$ to field $\mathcal{B}$, and $\sim$ indicates structural proportionality rather than precise quantitative equality.[23]

Definition 5 (**The Holistic Block Ethical Field Tensor**). *The three intra-dimensional tensors and their inter-dimensional couplings can be assembled into a single holistic block tensor $\mathscr{E}^{\mathcal{H}}$ over the combined space of all ethical sub-dimensions:*

$$\mathscr{E}^{\mathcal{H}}(w) = \begin{bmatrix} \mathcal{G}(w) & \mathcal{GR}(w) & \mathcal{GV}(w) \\ \mathcal{RG}(w) & \mathcal{R}(w) & \mathcal{RV}(w) \\ \mathcal{VG}(w) & \mathcal{VR}(w) & \mathcal{V}(w) \end{bmatrix}$$

where the diagonal blocks $\mathcal{G}(w)$, $\mathcal{R}(w)$, $\mathcal{V}(w)$ are the intra-dimensional tensors defined above, and the off-diagonal blocks $\mathcal{GR}(w)$, $\mathcal{GV}(w)$, $\mathcal{RG}(w)$, $\mathcal{RV}(w)$, $\mathcal{VG}(w)$, $\mathcal{VR}(w)$ encode the inter-dimensional couplings.[24]

The holistic block tensor $\mathscr{E}^{\mathcal{H}}(w)$ encodes the complete ethical field configuration at a case w: both the evaluative content within each dimension and the coupling between dimensions. It is a single mathematical object that unifies the three fundamental ethical dimensions without reducing any to the others, representing both their distinctness (they correspond to different blocks) and their interdependence (the off-diagonal blocks are generically non-zero). This is the ethical analog of the Faraday tensor's unification of the electric and magnetic fields, with the significant difference that the ethical tensor has a richer structure (three fundamental dimensions rather than two; general rather than antisymmetric; both intra-dimensional and inter-dimensional coupling) that reflects the greater complexity of the ethical domain.[25]

The framework is now in place. The next section draws out its philosophical consequences.

[23] The coupling operators $C^{\mathcal{A}\to\mathcal{B}}$ are, in full generality, tensor-valued maps from $\mathbb{R}^{n_{\mathcal{A}}\times n_{\mathcal{A}}}$ to $\mathbb{R}^{n_{\mathcal{B}}\times n_{\mathcal{B}}}$, and are themselves field-valued (i.e., they may vary across W). The product $\cdot$ denotes an appropriate tensorial product whose precise form is a task for substantive ethical theory. The multiplicative structure is philosophically motivated: it encodes the fact that each field's influence on a third is modulated by the configuration of the remaining field, rather than contributing independently. This nonlinear coupling is what generates the non-additive dynamics (virtuous and vicious spirals) discussed in §4.5: when all coupling contributions are positive, their product amplifies faster than their sum; when one contribution is severely degraded, the product structure propagates the damage more aggressively than an additive model would predict. The multiplicative form is also consonant with the nonabelian character of the ethical connection field argued for in §4.7.3, where products of field components (rather than sums) arise from the noncommutativity of the Lie algebra generators.

[24] The off-diagonal blocks are rectangular in general (since $n_{\mathcal{G}}$, $n_{\mathcal{R}}$, and $n_{\mathcal{V}}$ need not be equal), and the block tensor $\mathscr{E}^{\mathcal{H}}$ is an $(n_{\mathcal{G}} + n_{\mathcal{R}} + n_{\mathcal{V}}) \times (n_{\mathcal{G}} + n_{\mathcal{R}} + n_{\mathcal{V}})$ matrix. As with the intra-dimensional tensors, $\mathscr{E}^{\mathcal{H}}$ is general: $\mathscr{E}^{\mathcal{H}}_{ij}(w) \neq \mathscr{E}^{\mathcal{H}}_{ji}(w)$ in general, reflecting the directional asymmetry of both intra- and inter-dimensional coupling.

[25] The holistic block tensor $\mathscr{E}^{\mathcal{H}}$ also has definite transformation properties under changes of evaluative frame. When the coordinate system (evaluative vocabulary, calibration) changes, the components of $\mathscr{E}^{\mathcal{H}}$ transform covariantly, but the geometric object—the tensor itself—is invariant. This is precisely the frame-independence that the companion papers' invariance requirements demand: the ethical content encoded in $\mathscr{E}^{\mathcal{H}}$ is objective in the sense that it does not depend on the local representational conventions of any particular evaluator. The tensor formulation thus implements the objectivity-as-invariance principle at the level of ethical field content, not merely at the level of individual evaluative verdicts.

4 Upshots

The tensor-field framework developed in §3 is not merely a notational device. It generates philosophical consequences that are not available from within any single ethical tradition and that could not have been articulated without the formal apparatus. This section develops eight such consequences.

4.1 A non-reductive unification of the three dimensions

The tensor-field framework provides something that has been lacking in moral theory: a way of relating the axiological, deontic, and aretaic dimensions that is neither reductive nor merely pluralist.

Reductive strategies try to derive one or two of the dimensions from the remaining one: consequentialism derives the right and virtue from the good; strong deontology derives the good and virtue from the right; strong virtue ethics derives the good and the right from character. Each faces the difficulties rehearsed in §1.5.1. Pluralist strategies acknowledge all three dimensions as irreducible but offer no account of how they are related, leaving the moral landscape as a confederation of incommensurable domains.

The tensor-field model dissolves this dilemma. The three dimensions are unified in the sense that they are components of a single block tensor $\mathscr{E}^{\mathcal{H}}(w)$, dynamically coupled by inter-field coupling operators. But they remain irreducible in the sense that none is derivable from the others: the diagonal blocks $\mathcal{G}(w)$, $\mathcal{R}(w)$, and $\mathcal{V}(w)$ carry independent evaluative content that is not determined by the off-diagonal coupling blocks alone. This is precisely the kind of unification Maxwell achieved for electricity and magnetism: non-reductive, dynamic, and structural. The electric and magnetic fields are not separate entities that happen to co-occur (pluralism), and neither is "really" a manifestation of the other (reductionism). They are co-fundamental aspects of a single electromagnetic field whose dynamics are governed by their mutual coupling. The ethical case, I have argued, exhibits the same pattern.

The non-reductive character of the unification extends to the intra-dimensional level as well. Within each fundamental dimension, the tensor model preserves the irreducible plurality of sub-dimensions (welfare, autonomy, fairness, etc., within $\mathcal{G}$; non-maleficence, justice, fidelity, etc., within $\mathcal{R}$; courage, justice, temperance, etc., within $\mathcal{V}$) while representing their couplings in a single mathematical object. This means the framework can speak simultaneously to debates *within* each tradition (e.g., debates among consequentialists about how different dimensions of welfare trade off) and to debates *between* traditions (about which fundamental dimension is primary). That is considerable philosophical range from a single formal framework.

4.2 A formal model of the ethical landscape

The tensor-field model provides, for the first time, a formal representation of the ethical landscape that captures both the multidimensionality and the coupling structure of ethical evaluation. This contribution has value independent of the electromagnetic analogy: anyone who accepts that ethical evaluation is multidimensional and that its dimensions are

interdependent—and it is difficult to see how one could deny either claim—should welcome a formal framework that represents both features.

The point can be put sharply. Before the present framework, formal ethics had two broad options for representing the evaluative content of a moral situation. It could use a scalar (a single number, e.g., "utility"), which discards all structural information about how different evaluative considerations relate. Or it could use a vector (a list of numbers, one per dimension), which preserves multidimensionality but discards all information about coupling. The tensor is the simplest mathematical object that preserves *both*: the distinctness of the sub-dimensions (diagonal entries) and the pattern of their relationships (off-diagonal entries). This is not a matter of mathematical taste. It is a matter of representational adequacy: the ethical reality has coupling structure, and a formal model that discards it is leaving out information that is, as the following subsections demonstrate, philosophically essential.[26]

4.3 Illuminating the structure of moral disagreement

The tensor-field framework reframes the classical disagreements among ethical traditions as structural hypotheses about the ethical field tensor. This does not dissolve the disagreements, but it does make them more precise and, in some cases, reveals that they are less fundamental than they appear.

Consider. Consequentialism, in the tensor-field framework, corresponds to the claim that the axiological diagonal block $\mathcal{G}(w)$ is explanatorily primary and that the deontic and aretaic content is derivable from the axiological content via the coupling operators: the off-diagonal blocks $\mathcal{GR}$ and $\mathcal{GV}$ are held to be "large" (strong coupling from $\mathcal{G}$ to the other fields), while the reverse couplings $\mathcal{RG}$ and $\mathcal{VG}$ are held to be negligible or derivative. Deontology corresponds to the analogous claim about $\mathcal{R}$. Virtue ethics corresponds to the analogous claim about $\mathcal{V}$.

These are not incompatible *metaphysical* claims about the nature of morality. They are different hypotheses about the *relative magnitudes* of the diagonal and off-diagonal components of the ethical field tensor—hypotheses that can, at least in principle, be compared structurally. The consequentialist who claims that the good is explanatorily primary is making a claim about the coupling structure: that the $\mathcal{G} \to \mathcal{R}$ and $\mathcal{G} \to \mathcal{V}$ couplings are strong enough to determine the content of $\mathcal{R}$ and $\mathcal{V}$, while the reverse couplings are weak. The deontologist denies this, claiming instead that $\mathcal{R}$ exerts constitutive influence on $\mathcal{G}$ that cannot be reduced to axiological feedback.

This reframing has a clarifying effect. It shows that the disagreements among ethical traditions are not necessarily all-or-nothing: a nuanced position might hold that, in some regions of the ethical landscape, the $\mathcal{G} \to \mathcal{R}$ coupling dominates (consequentialism gets things roughly right in those cases), while in other regions, the $\mathcal{R} \to \mathcal{G}$ coupling dominates

[26] An analogy may help. Consider the difference between knowing the temperature at each point in a room (a scalar field) and knowing the full stress state at each point in an elastic body (a tensor field). The scalar tells you how hot each point is; it does not tell you how the heat at one point relates to the heat at neighboring points, or how temperature differences generate thermal stresses. The tensor tells you not only the magnitudes of the stresses but how they couple across directions. A scalar description of an elastic body under load would miss most of the mechanically significant information. Similarly, a scalar description of the ethical landscape misses most of the ethically significant information.

(deontology is more illuminating there). The framework provides a vocabulary for expressing such mixed verdicts without incoherence. It also opens the possibility that the classical traditions are not so much wrong as *partial*: each captures the structure of the ethical field in the regions where its preferred dimension's couplings dominate, and each mischaracterizes the structure in regions where they do not.[27]

4.4 The structure of ethical dilemmas

The tensor framework provides a structural characterization of genuine ethical dilemmas that distinguishes them from merely difficult choices.

A difficult choice involves competing considerations that are, in principle, commensurable: one must weigh the costs and benefits along a single dimension, or across independent dimensions, and select the best available option. The difficulty is epistemic or practical (one may not know the magnitudes, or the best option may be costly), but the evaluative structure does not *itself* generate the conflict.

A genuine ethical dilemma, by contrast, is a situation in which the evaluative structure generates an irresolvable tension. In the tensor framework, this has a precise characterization: a genuine dilemma is a situation in which the off-diagonal entries of the relevant tensor are significantly negative, where promoting one sub-dimension *constitutively* degrades another, not because of external scarcity or practical limitation but because of the internal coupling structure of the ethical field at that point. The conflict is not between independent considerations that happen to compete for limited resources; it is between considerations whose coupling is antagonistic.

Consider the off-diagonal entries of $\mathcal{R}(w)$ in a situation where a physician must decide whether to override a patient's stated preferences in order to save their life. The diagonal entries of $\mathcal{R}$ may both be large and positive (the duty of beneficence is strong; the duty to respect autonomy is strong). But the coupling entry $\mathcal{R}_{45}(w)$ is strongly negative: in this situation, fulfilling beneficence undermines respect for autonomy, and respecting autonomy undermines beneficence. The dilemma is *structural*: it lives in the off-diagonal entry, not in the diagonal entries. No amount of additional information about the magnitudes of the diagonal entries can resolve it, because the conflict is generated by the coupling itself.

This gives formal expression to what moral philosophers from the Greek tragedians through Bernard Williams and Martha Nussbaum have recognized: that some moral conflicts are not merely hard but structurally irresolvable, and that their irresolvability is a feature of the ethical landscape, not a deficiency of the moral reasoner.[28]

[27] This is a version of the familiar "grain of truth" intuition about ethical theories—the sense that consequentialism, deontology, and virtue ethics each get something right. The tensor-field framework gives this intuition formal substance: each tradition correctly identifies the coupling structure in a particular region of the ethical landscape and incorrectly generalizes from that region to the whole.

[28] Williams (1965) and Nussbaum (1986) both argue, in different ways, that genuine moral dilemmas are not artifacts of incomplete moral theories but reflections of irreducible features of the moral domain. The tensor framework gives this insight a formal home: a dilemma is a region of the ethical landscape where the off-diagonal entries of one or more ethical tensors are significantly negative. The framework does not eliminate dilemmas (it cannot, since the coupling structure is a feature of the ethical reality, not of the model); it characterizes them structurally and distinguishes them from superficially similar but structurally different kinds of moral difficulty.

The characterization extends naturally to *cross-dimensional* dilemmas: situations where the inter-field coupling is antagonistic. A case in which doing the right thing ($\mathcal{R}$) requires sacrificing significant goods ($\mathcal{G}$), or in which virtuous action ($\mathcal{V}$) conflicts with what duty requires ($\mathcal{R}$), is a case where the off-diagonal blocks of the full ethical tensor $\mathscr{E}^{\mathcal{H}}(w)$ contain negative entries. The framework thus provides a unified treatment of both intra-dimensional dilemmas (conflicts within the good, or within the right, or within virtue) and inter-dimensional dilemmas (conflicts between the good and the right, or between the right and virtue, or between virtue and the good).

4.5 Moral synergy

The tensor framework provides an equally precise characterization of the opposite of phenomenon: moral synergy.

Moral synergy is the phenomenon whereby promoting one ethical consideration simultaneously promotes others, i.e., where doing the right thing also produces good outcomes and exemplifies virtue, or where cultivating a virtue both enhances well-being and strengthens the disposition to act rightly. In the tensor framework, moral synergy corresponds to positive off-diagonal entries: situations in which the coupling between sub-dimensions, or between fundamental ethical dimensions, is reinforcing rather than antagonistic.

A case of intra-dimensional synergy within $\mathcal{G}$: a social policy that enhances welfare ($\mathcal{G}_{11}$ positive) *by* expanding autonomy ($\mathcal{G}_{22}$ positive), where the mechanism of welfare-promotion is autonomy-enhancing ($\mathcal{G}_{12}$ and $\mathcal{G}_{21}$ both positive). The off-diagonal entries tell us something the diagonal entries cannot: that the goods are not merely co-present but *mutually enabling*. The policy is better than the sum of its parts.

A case of inter-dimensional synergy: an act of courageous truth-telling that fulfills a duty of honesty ($\mathcal{R}$), promotes the good of the community by enabling informed collective decision-making ($\mathcal{G}$), and exemplifies and strengthens the agent's virtues of courage and integrity ($\mathcal{V}$). The off-diagonal blocks $\mathcal{GR}$, $\mathcal{RV}$, and $\mathcal{VG}$ are all positive: each dimension reinforces the others. The ethical significance of the act exceeds what any single dimension can capture.

The existence, character, and conditions of moral synergy are substantive ethical questions that the tensor framework makes formally tractable. Under what conditions do the off-diagonal entries tend to be positive? Are there structural features of moral situations, institutions, or character that promote synergy? The Aristotelian tradition has long maintained that the virtues are mutually supporting and jointly constitutive of flourishing, a claim that, in the tensor framework, amounts to the hypothesis that $\mathcal{V}_{ij} \geq 0$ for all virtue-pairs and that the inter-field couplings $\mathcal{VG}$ and $\mathcal{VR}$ are generally positive for well-cultivated character. Whether this hypothesis is correct is an open question; that it can now be stated with structural precision is an advance.

4.6 Non-additive ethical patterns: virtuous and vicious spirals

Because the three fields $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ are dynamically coupled, their interactions can generate *non-additive* patterns, i.e., emergent ethical structures that cannot be predicted from any single dimension in isolation. The most striking of these are self-amplifying spirals.

4.6.1 Virtuous spirals

When the inter-field coupling is positively reinforcing across all three dimensions, a self-amplifying upward dynamic can emerge. Consider: mutual benefit ($\mathcal{G}$ positive) fosters a disposition to do right by one another ($\mathcal{R}$ positive), which cultivates virtuous dispositions of trust, generosity, and cooperativeness ($\mathcal{V}$ positive), which in turn expand the horizon of mutual benefit (further positive change in $\mathcal{G}$). The off-diagonal coupling terms are all positive and mutually reinforcing: $C^{\mathcal{G}\to\mathcal{R}}[\mathcal{G}] > 0$, $C^{\mathcal{R}\to\mathcal{V}}[\mathcal{R}] > 0$, $C^{\mathcal{V}\to\mathcal{G}}[\mathcal{V}] > 0$. Each positive shift in one field generates a positive shift in the next, which feeds back to amplify the first.

These spirals are familiar from the best cases of moral community: deep friendships, well-functioning institutions, and societies in which things are "going well" in a way that seems to be more than the sum of its parts. The tensor-field model explains *why* it is more than the sum: the positive off-diagonal coupling terms generate evaluative content that is not reducible to the diagonal entries alone. The "excess" goodness, rightness, and virtue of a flourishing community is not a mysterious emergent property; it is the contribution of the coupling terms, which are as much a part of the ethical field configuration as the diagonal entries are.

There is an instructive parallel with the electromagnetic case. Electromagnetic waves, i.e., light, are the emergent product of the dynamic coupling between the electric and magnetic fields: a changing electric field generates a changing magnetic field, which generates a changing electric field, in a self-sustaining oscillation. Light is not an electric phenomenon or a magnetic phenomenon; it is an electromagnetic phenomenon, a product of the coupling. Virtuous spirals are, structurally, the ethical analog of electromagnetic waves: self-sustaining patterns generated by the positive feedback among dynamically coupled fields. Just as light cannot exist without the coupling between $\mathscr{E}$ and $\mathbf{B}$, virtuous spirals cannot exist without the coupling among $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$.

4.6.2 Vicious spirals

The converse dynamic is equally real and equally illuminating. When the inter-field coupling is negatively reinforcing, a self-amplifying downward spiral can emerge: mutual harm ($\mathcal{G}$ negative) erodes the sense of obligation and disposes agents toward wrongdoing ($\mathcal{R}$ negative), which corrupts character and cultivates vices of distrust, cruelty, and exploitation ($\mathcal{V}$ negative), which deepens and entrenches the patterns of mutual harm (further negative change in $\mathcal{G}$). Each degradation in one field drives degradation in the others, and the cycle feeds on itself.

These vicious spirals are familiar from the worst cases of moral breakdown—cycles of injustice, institutional decay, and moral corruption that seem to feed on themselves and resist piecemeal intervention. The tensor-field model provides a structural explanation of their self-sustaining character. A purely axiological theory (consequentialism) can describe the harms but cannot model the feedback through duty and character that sustains them. A purely deontic theory can describe the violations of obligation but cannot model the feedback through value and virtue. A purely aretaic theory can describe the corruption of character but cannot model the feedback through value and obligation. Only a framework that represents the coupling among all three fields can capture the full dynamic.

4.6.3 The structural character of spiraling dynamics

The virtuous and vicious spirals just described have a common structural feature: they are *emergent properties of the coupling*, not features of any single field in isolation. The tensor-field framework identifies precisely what makes them possible (the sign and magnitude of the inter-field coupling terms) and thereby makes the conditions for their emergence, persistence, and disruption formally tractable. This has immediate consequences for moral practice: if one wishes to initiate a virtuous spiral, one should attend not only to the diagonal entries of the individual fields (promoting good outcomes, fulfilling duties, cultivating virtues) but to the coupling structure itself, i.e., to the conditions under which improvements in one dimension reinforce improvements in the others.[29]

4.7 Syntegrity: the geometry of ethical flourishing

The spiraling dynamics of the previous subsection arise from the *sign structure* of the inter-field coupling: positive coupling generates virtuous spirals; negative coupling generates vicious ones. This naturally raises a question: is there an *ideal* configuration of the ethical field tensor, a configuration toward which virtuous spirals tend and from which vicious spirals depart?

The tensor framework suggests that there is. I propose to call it **syntegrity**: the condition in which the ethical field tensor $\mathscr{E}^{\mathcal{H}}(w)$ exhibits positive values in every diagonal entry and synergistic (positive) values in every off-diagonal entry, at both the intra-dimensional and inter-dimensional levels. The term is intended to capture the convergence of several structural features: the *symmetry* of the condition (it applies uniformly across all three ethical dimensions and all their sub-dimensions, privileging none), the *synergy* it encodes (every coupling is mutually reinforcing), the *symbiosis* it represents (the three dimensions do not merely coexist but actively sustain one another), and the *integrity* of the whole (the ethical field configuration hangs together as a unified, self-reinforcing structure that goes beyond mere coherence to encompass the full richness of ethical life).

More precisely, syntegrity obtains at a case $w \in W$ when:

1. Every diagonal entry of each intra-dimensional tensor is sufficiently positive: $\mathcal{G}_{ii}(w) > 0$ for all i, $\mathcal{R}_{ii}(w) > 0$ for all i, $\mathcal{V}_{ii}(w) > 0$ for all i. All relevant sub-dimensions of goodness, rightness, and virtue are actively realized.

2. Every off-diagonal entry of each intra-dimensional tensor is positive: $\mathcal{G}_{ij}(w) > 0$, $\mathcal{R}_{ij}(w) > 0$, $\mathcal{V}_{ij}(w) > 0$ for all $i \neq j$. The sub-dimensions within each ethical field are mutually reinforcing—welfare enhances autonomy, justice supports fidelity, courage strengthens practical wisdom, and so on in every direction.

3. Every off-diagonal block of the holistic block tensor $\mathscr{E}^{\mathcal{H}}(w)$ has positive entries. The three fundamental ethical dimensions are mutually reinforcing: promoting the good

[29] This point has practical implications for institutional design, social policy, and moral education. An institution that promotes good outcomes ($\mathcal{G}$ positive) but does so through mechanisms that corrupt character ($C^{\mathcal{G}\to\mathcal{V}} < 0$—e.g., incentive structures that reward results through morally corrosive means) may achieve short-term axiological gains at the cost of long-term vicious spiraling. The tensor-field framework makes this risk structurally visible.

also promotes right action and virtuous character; fulfilling duty also promotes genuine goods and cultivates virtue; exercising virtue also realizes goods and fulfills obligations.

Syntegrity is a *formal* ideal: it specifies the abstract geometry of a holistically flourishing ethical configuration—the shape that the ethical field must have for all evaluative dimensions to be realized and all couplings to be harmonious—while leaving entirely open which concrete configurations of agents, institutions, practices, and material conditions *source* this field state. Many possible configurations can satisfy the syntegrity conditions, just as many different physical structures (a bridge, a bone, a cathedral vault) can share the same stress-tensor profile. The ideal constrains the geometry without dictating the architecture. This is a strength: it means syntegrity is ecumenical with respect to substantive ethical content. A Kantian community organized around duty, a consequentialist community organized around welfare, and an Aristotelian community organized around virtue could each, in principle, achieve syntegrity—provided that their preferred organizational principle generates positive coupling to the other two dimensions rather than antagonizing them. The ideal does not favor one tradition; it requires each to take the other dimensions seriously.

4.7.1 Syntegrity and the Stoic sage

The formal ideal of syntegrity has a striking resonance with one of the oldest and most demanding ideals in the history of ethics: the Stoic conception of the sage (*sophos*).

The Stoic sage, as characterized by Chrysippus, Epictetus, and others, is distinguished not by excelling along a single ethical dimension but by an extraordinary *comprehensiveness*: the sage draws genuine benefit (axiological) from every situation, acts correctly (deontic) in every circumstance, and does so from a character that is fully virtuous (aretaic), and, crucially, these three achievements are not independent. The sage does not happen to be simultaneously good, right-acting, and virtuous, as though hitting three separate targets at once. Rather, the sage's virtue *constitutes* the capacity to recognize and realize genuine goods, which in turn *constitutes* the capacity to discern and fulfill obligations, which in turn sustains and deepens the sage's virtue. The three dimensions, in the sage, are seamlessly and synergistically integrated.

In the tensor-field framework, this is precisely the condition of syntegrity evaluated at the level of an individual agent's ethical field configuration. The sage is an agent for whom $\mathscr{E}^{\mathcal{H}}(w)$ exhibits syntegrity across the full range of ethically relevant cases: every diagonal entry positive, every off-diagonal entry positive, at both the intra-dimensional and inter-dimensional levels. The sage's practical wisdom (which the Stoics, like Aristotle, regarded as the architectonic virtue) can be understood, in tensor terms, as the capacity that ensures positive off-diagonal coupling throughout $\mathcal{V}$ and across the inter-field blocks. Practical wisdom is what makes the sage's courage also just, the sage's justice also compassionate, and the sage's compassion also conducive to genuine flourishing. It is, in the language of §4.8, the virtue with large positive off-diagonal entries to all others, and the tensor framework now reveals that its significance extends beyond the aretaic dimension: it is the capacity that sustains syntegrity across the entire ethical field.[30]

[30]The Stoic version of the thesis of the unity of the virtues (that one cannot possess any virtue fully without possessing all of them) corresponds, in the tensor framework, to the claim that syntegrity within the aretaic

Kant engages directly with the Stoic ideal. In the *Critique of Practical Reason* and the *Religion within the Boundaries of Mere Reason*, he treats the idea of complete moral perfection (the "holy will" or the moral archetype) as a standard that finite rational agents can approach but never fully instantiate. The archetype is not a description of any actual agent but a *regulative ideal*: it specifies the direction of moral progress and provides the measuring stick against which actual moral achievement is assessed.[31]

The tensor-field framework gives these ancient and modern ideals a new formal articulation. Syntegrity is the Stoic sage's ethical field configuration rendered as a precise mathematical condition on the holistic block tensor $\mathscr{E}^{\mathcal{H}}$. Its value as an ideal does not depend on whether any actual agent or community fully instantiates it. What it provides is a *formal characterization of the target*: the geometry of the ethical field toward which moral progress is directed. This turns the regulative ideal from an informal aspiration ("be like the sage") into a formally specifiable structural condition on the ethical field tensor, one that can be investigated, approximated, partially satisfied, and used as a diagnostic: given a community's or agent's actual field configuration, how far is it from syntegrity, and which entries—which couplings, which dimensions—are responsible for the shortfall?[32]

4.7.2 Syntegrity as attractor and the dynamics of moral progress

The spiral dynamics of §4.5 suggest a further structural point about syntegrity: it functions as an *attractor* of the coupled ethical dynamics. When the off-diagonal coupling terms are positive, the virtuous spiral drives the ethical field configuration toward ever-greater realization of all dimensions and ever-stronger positive coupling among them, i.e., toward syntegrity. The syntegral configuration is the state toward which virtuous spirals converge. Conversely, the antisyntegral configuration (in which couplings are predominantly negative and dimensional values are degraded) is the attractor of vicious spirals.

This gives a dynamical interpretation of moral progress that complements the regulative interpretation drawn from Kant. Moral progress is not merely movement toward a static ideal specified from outside the dynamics; it is the *natural trajectory* of an ethical system

tensor $\mathcal{V}$ is an all-or-nothing condition: a configuration in which some off-diagonal entries of $\mathcal{V}$ are positive and others negative is not a case of "partial virtue" but of a fundamentally different (non-syntegral) field configuration. Whether this strong Stoic thesis is correct is an open question; the tensor framework makes the thesis precise enough to investigate.

[31] Kant, *Critique of Practical Reason*, 5:32–33, and *Religion within the Boundaries of Mere Reason*, 6:61–62. Kant's moral archetype shares the Stoic sage's comprehensiveness but differs in its relationship to happiness: for the Stoics, virtue is sufficient for eudaimonia; for Kant, virtue makes one *worthy* of happiness, but happiness itself depends on conditions outside the agent's control. In the tensor-field framework, this disagreement can be located precisely: it concerns whether syntegrity within $\mathcal{V}$ *necessitates* syntegrity within $\mathcal{G}$ (the Stoic view) or merely generates strong positive coupling toward it (the Kantian view, which allows for external contingencies to block the axiological realization even when the aretaic conditions are met).

[32] This diagnostic capacity connects syntegrity to the applied dimensions of the broader research program. In institutional design, for instance, syntegrity serves as a design target: an institution whose characteristic ethical situations exhibit syntegrity is one in which doing the right thing, producing good outcomes, and cultivating virtuous character mutually reinforce one another. The distance of an institution's actual ethical field configuration from the syntegrity ideal provides a formal measure of its ethical health, one that is more structurally informative than any single-dimensional metric (e.g., compliance rates, welfare measures, or character assessments in isolation) could be.

whose coupling structure is positive. A community that manages to achieve positive inter-field coupling in its institutions, practices, and character formation will, by the logic of the coupled dynamics, tend toward syntegrity, not because syntegrity has been imposed as an external target but because it is the dynamical equilibrium of the positively coupled system.

This also illuminates what makes moral regress so difficult to reverse. A community caught in a vicious spiral is not merely failing to make progress toward an ideal; it is being pulled by the logic of its own coupling structure toward an antisyntegral attractor. Reversing the spiral requires not merely improving one dimension in isolation (which the multiplicative coupling structure of Definition 4 predicts will have limited effect if the other fields remain degraded) but *restructuring the coupling itself*, i.e., turning negative off-diagonal entries positive, so that improvements in one dimension begin to reinforce rather than undermine the others. The tensor-field framework identifies this as the structural task of moral repair: not merely treating symptoms (negative diagonal entries) but restoring the synergistic coupling (positive off-diagonal entries) that enables virtuous spiraling toward syntegrity.

4.7.3 Visualizing syntegrity: the ethical Birkeland current

The formal conditions on the tensor $\mathscr{E}^{\mathcal{H}}(w)$ that define syntegrity (positive diagonals, positive off-diagonals, positive inter-field blocks) are abstract. It is worth pausing to ask whether there is a concrete dynamical image that makes the abstract conditions intuitive: a picture of what syntegrity *looks like* when the tensor conditions are dynamically embodied in the coupled evolution of the three ethical fields.

I suggest that such an image is provided by a structural analogy with a phenomenon in plasma physics: the Birkeland current.

A Birkeland current is a self-organizing plasma structure in which an electric current flows along a filamentary axis while a magnetic field circulates around it, and the resulting coupled dynamics sustain a coherent, stable, self-reinforcing configuration. The structure is not imposed from outside; it emerges from the dynamic coupling between the electric and magnetic fields within the plasma. Three features of the Birkeland current are relevant.[33]

First, the *circulating magnetic field*. In a Birkeland current, the magnetic field lines form closed loops around the current axis. This circulating structure provides confinement: it holds the plasma together, preventing the charged particles from dispersing. Without the circulating magnetic field, the current would dissipate; the filament would lose its coherence. The circulation is what binds the constituents of the plasma into a sustained relational structure.

Second, the *directed electric current*. The current flows along the axis of the filament, guided and sustained by the magnetic confinement. But the relationship is not one-way: the current *generates* the magnetic field that confines it. The directed current and the circulating magnetic field are co-constitutive—each sustains the other. The confinement is not the work of the magnetic field alone, nor of the current alone; it is the joint product of their dynamic

[33] For a classical treatment of Birkeland currents and the plasma physics of field-aligned currents, see, e.g., Alfvén (1981). The analogy developed here is structural: the claim is not that ethical dynamics are plasma dynamics but that the pattern of coupled self-organization in a Birkeland current—three dynamically intertwined aspects forming a self-sustaining whole—maps onto the pattern of coupled self-organization in a syntegral ethical configuration.

coupling. Remove either, and the Birkeland current collapses.

Third, the *helical, self-amplifying dynamics*. Within the confined structure, the charged particles trace helical paths around the axis, spiraling as they move along the filament. This helical motion is the visible expression of the full coupled dynamics: the spiral is sustained by the confinement, and the confinement is sustained by the coherent motion of the spiraling charges. The result is a self-organizing, self-sustaining filamentary structure whose stability depends on the mutual reinforcement of all three aspects: circulation, direction, and spiral.

The ethical analog maps onto this tripartite structure with genuine precision. The three ethical fields—$\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$—play roles that correspond to the three aspects of the Birkeland current, and their coupled dynamics exhibit the same pattern of co-constitutive mutual sustenance.

Circulating mutual benefit ($\mathcal{G}$). The axiological field, in a syntegral configuration, takes the form of circulating patterns of mutual benefit: goods that flow between agents, sustaining relationships of shared stake, mutual dependence, and reciprocal care. This circulating structure provides one essential component of ethical confinement: it *binds* agents together in relations of mutual benefit that give them a sustained reason to remain in ethical relationship with one another. Just as the circulating magnetic field holds the plasma together by preventing dispersion, the circulating mutual benefit holds a moral community together by sustaining the relational bonds on which the other ethical dynamics depend. Without circulating mutual benefit, i.e., in a community where interactions are zero-sum or mutually harmful, agents have no sustained stake in one another's flourishing, and the ethical filament loses its cohesion.

Reflecting right ($\mathcal{R}$). The deontic field, in a syntegral configuration, takes the form of reflecting patterns of reciprocal obligation: rights and duties that flow between agents and return to them, creating a structure of mutual accountability. Rights are inherently relational and reciprocal: my right generates your duty, your right generates mine, and the deontic content reflects between the agents who stand in these relations. This reflecting structure provides the second essential component of ethical confinement: it gives the relational bonds their *normative scaffolding*, ensuring that the circulating mutual benefit is not merely fortunate coincidence but is sustained by a structure of obligations that agents can recognize, invoke, and hold one another to. Just as the directed current generates and sustains the magnetic confinement while being sustained by it in return, the reflecting deontic structure generates and sustains the axiological confinement (by shaping which goods are achievable within the bounds of right) while being sustained by it in return (the axiological landscape of mutual benefit provides the material conditions within which obligations can be recognized and fulfilled).

The confinement is thus, as in the physical case, *co-constitutive*: neither the circulating good nor the reflecting right alone suffices. Without reflecting rights, circulating mutual benefit has no normative structure; it is a happy accident rather than an ethical achievement, vulnerable to disruption by any agent who finds it advantageous to defect. Without circulating mutual benefit, reflecting rights have no material substrate; they are formal demands with nothing to sustain them, obligations that agents lack the resources or the relational bonds to fulfill. The ethical confinement that makes a syntegral configuration stable is the joint product of $\mathcal{G}$ and $\mathcal{R}$ dynamically sustaining each other, just as the physical confinement of a Birkeland current is the joint product of the magnetic field and the electric current

dynamically sustaining each other.

Spiraling virtue ($\mathcal{V}$). Within the ethical confinement jointly constituted by circulating mutual benefit and reflecting rights, the aretaic field takes the form of spiraling dynamics of character development: virtues that wind upward (in the syntegral case) as the confinement sustains and nourishes their cultivation, or vices that wind downward (in the antisyntegral case) as the breakdown of confinement degrades and corrupts character. The helical, spiraling character of the aretaic dynamics—the fact that virtues (or vices) do not merely persist but amplify, deepening with each cycle of the coupled dynamics—corresponds to the helical motion of charged particles in a Birkeland current: the spiral is sustained by the confinement, and the coherent spiraling motion is itself part of what sustains the confinement. In the ethical case, the cultivation of virtue strengthens both the circulating mutual benefit (virtuous agents expand the horizon of achievable goods) and the reflecting rights (virtuous agents are more reliably responsive to obligations and more perceptive of the obligations that obtain), thereby reinforcing the confinement that sustains the very conditions under which virtue can be cultivated. This is the ethical analog of the plasma's helical dynamics sustaining the current that generates the magnetic field that confines the plasma in which the helical dynamics occur: a self-sustaining loop whose stability is the stability of syntegrity itself.

The Birkeland current image thus makes visible several features of syntegrity that the abstract tensor conditions, by themselves, do not convey.

First, the *co-constitutive* character of ethical confinement: syntegral stability is not the work of any single dimension but the joint product of the circulating good and the reflecting right, each sustaining the other. This is why intervening on a single dimension in isolation (improving outcomes without attending to the structure of rights, or enforcing rights without attending to whether mutual benefit circulates) is insufficient for achieving or maintaining syntegrity: both components of confinement must be in place.

Second, the *filamentary* character of syntegral structures: a syntegral community is not a diffuse state of general well-being but a *coherent structure*, a filament of mutually reinforcing ethical dynamics that holds together because its internal coupling is positive. Like a Birkeland current, it has a definite shape, a definite internal organization, and a definite boundary between the coherent interior and the unconfined exterior.

Third, the *self-organizing* character of syntegral dynamics: the syntegral configuration is not imposed from outside but emerges from the dynamic coupling among the three fields. When the coupling is positive, the system organizes itself into a coherent filament; when the coupling is negative, the system disorganizes into the ethical analog of a dissipating plasma. The Birkeland current image makes vivid what the attractor analysis of the preceding subsection established formally: syntegrity is the natural equilibrium of a positively coupled ethical system, not a target imposed by external authority.

Fourth, the *vulnerability* of syntegral structures: a Birkeland current can be disrupted by sufficiently strong external perturbation (a cross-field disturbance that breaks the confinement and causes the filament to dissipate). Similarly, a syntegral ethical configuration can be disrupted by sufficiently severe shocks to the axiological or deontic confinement: a catastrophic harm that ruptures the circulating mutual benefit, or a gross injustice that shatters the reflecting structure of rights. Once the confinement is broken, the spiraling virtue that depended on it unravels, and the system may be captured by the vicious attractor described

in §4.5.2. The Birkeland current image thus provides not only a picture of syntegral flourishing but also a picture of how syntegral flourishing can be lost (and, by implication, of what must be protected if it is to be sustained).[34]

4.8 Connection to the invariance framework and gauge-theoretic structure

The tensor-field model connects to the formal foundations developed in the companion papers in two ways that are worth making explicit.

4.8.1 Implementing local invariance at the level of field content

The companion papers develop objectivity-as-invariance (OI) as a bridge principle linking the structure of practical reason to substantive ethical constraints, and they show that the demand for local invariance necessitates a connection field (identified, in the Geometry paper, with the Kantian good will) that enforces coherence across locally varying evaluative frames.[35]

The rank-2 character of the ethical tensor fields connects directly to these invariance requirements. A tensor is not merely a convenient way to package numbers; it is a mathematical object with definite *transformation properties*. When one changes coordinates—in the ethical case, when one changes the evaluative frame (the vocabulary, calibration, or conceptual scheme) through which ethical content is represented—the components of the tensor change, but the tensor itself, as a geometric object, is invariant. The diagonal entries may shift, the off-diagonal entries may redistribute, but the total evaluative content encoded in the tensor is preserved.

This is precisely what the companion papers' local invariance requirement demands: that the ethical content of a situation be frame-independent, even though its component representation varies across evaluative vocabularies. The tensor formulation implements this requirement at the level of ethical field content, not merely at the level of individual evaluative verdicts. It ensures that the coupling structure of the ethical landscape (the pattern of synergies and antagonisms among ethical sub-dimensions) is an objective feature of the ethical reality, not an artifact of any particular evaluative vocabulary.[36]

[34] The Birkeland current analogy also suggests a structural account of the *repair* of a disrupted ethical configuration. In plasma physics, a disrupted Birkeland current can re-form if the conditions for self-organization are restored: if the current is re-established and the magnetic confinement regenerates around it. In the ethical case, restoring a disrupted syntegral configuration requires re-establishing both components of ethical confinement simultaneously: restoring patterns of mutual benefit ($\mathcal{G}$ positive, with positive off-diagonal coupling) *and* rebuilding the structure of mutual obligation and accountability ($\mathcal{R}$ positive, with positive off-diagonal coupling). Only when both are in place can the spiraling virtue ($\mathcal{V}$) resume its upward helical dynamics. This is why piecemeal moral repair—addressing harms without restoring rights, or enforcing rights without rebuilding mutual benefit—so often fails to achieve lasting ethical recovery: it restores one component of confinement while leaving the other broken, and the partially confined system cannot sustain the coherent spiraling dynamics that syntegrity requires.

[35] Sanchez Borboa (manuscripts A and B). The Symmetry paper develops OI and the bootstrapping from global to local invariance; the Geometry paper develops the gauge-theoretic apparatus and identifies the connection field with the good will.

[36] In more technical terms: the tensor fields $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ are sections of tensor bundles over the ethical case

4.8.2 Specifying the ethical matter fields

The Geometry paper introduces "ethical matter fields" as the substantive considerations (welfare, rights, relationships, virtues) that the connection (good will) organizes into frame-independent evaluations. The footnote in that paper notes that the present paper provides a further specification of what these matter fields look like.[37]

The present paper fulfills that promissory note. The ethical matter fields are not unstructured quantities that the connection simply "acts on." They are rank-2 tensor fields with rich internal structure: diagonal entries encoding dimensional content, off-diagonal entries encoding coupling, and definite transformation properties that ensure covariance under local re-framings. The connection organizes this structured content into frame-independent evaluations; the tensor fields provide the content that the connection organizes. Together, the companion papers (which characterize the form: connection $\mathcal{C}$ and curvature/field strength $\mathcal{E}$) and the present paper (which characterizes the matter: the ethical tensor fields $\mathcal{R}, \mathcal{G}, \mathcal{V}$ and their couplings, represented in holistic ethical block field $\mathscr{E}^{\mathcal{H}}$) provide a complete picture in which both the form and the substance of the ethical landscape are formally specified.

This form/matter complementarity has an illuminating resonance. The Geometry paper shows that the good will (the connection) is the structure that makes objective ethical evaluation possible across locally varying frames. The present paper shows that the content being evaluated—the landscape of goods, duties, and virtues and their interrelations—has the structure of dynamically coupled rank-2 tensor fields. Neither paper is complete without the other: the form without the matter would be a connection with nothing to organize; the matter without the form would be ethical content with no guarantee of frame-independent objectivity. The two together constitute the full field-theoretic picture of the ethical domain.

4.8.3 How matter illuminates form

The two preceding subsections describe the relationship between the present paper and the companion papers as one of complementarity: the companion papers characterize the form (the ethical connection field), the present paper characterizes the matter (the ethical tensor fields), and each is incomplete without the other. But the relationship runs deeper than complementarity. When the form and the matter are brought together, the matter-field structure *reveals internal structure of the connection field itself*, structure that the Geometry paper's derivation (precisely because it works at the level of formal requirements on objective evaluation) does not by itself make visible.

The constitutive interdimensionality of the good will. Begin with a fact about Kant's own characterization of the good will that is so familiar that its structural significance is easy to miss. The good will, as Kant describes it, is simultaneously:

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space W, and the connection field (good will) developed in the Geometry paper acts on these bundles via covariant derivative, enforcing the frame-independence of parallel-transported ethical content. The coupling operators $C^{\mathcal{A}\to\mathcal{B}}$ must themselves be covariant, i.e., they must commute appropriately with the connection's action. The full development of this interaction between the connection and the ethical tensor fields is a task for future work; what matters here is that the tensor-field model is structurally compatible with, and naturally situated within, the gauge-theoretic framework.

[37] Sanchez Borboa (manuscript B), footnote 21.

- *Deontic*: it is defined by its orientation toward the moral law. The good will is the will that acts from duty, from the categorical imperative. Its constitutive relationship to the moral law is a relationship to the structure of obligation.

- *Aretaic*: it *is* moral worth. To have a good will is not merely to comply with duty externally but to act *from* duty, and this "acting from" is precisely what Kant means by moral worth. The good will is the fundamental feature of virtuous character, the *Gesinnung* (fundamental moral orientation) from which specific virtues derive their moral significance.

- *Axiological*: it is the only thing good "without limitation." The Geometry paper gives this claim a precise formal characterization (the good will's value is gauge-invariant) but the claim itself is an axiological claim. The good will has unconditional *value*; it is not merely required (deontic) or admirable (aretaic) but genuinely, irreducibly *good*.

These three aspects of the good will are typically treated as three independent philosophical claims that Kant happens to make about a single object. The tensor-field framework suggests a different and, I believe, deeper reading: they are three aspects of a single structural fact about the ethical connection field, one that becomes visible only when the connection is considered in relation to the matter fields it organizes.

Why the connection must be ethically interdimensional. The Geometry paper derives the ethical connection field from the requirement of local invariance: if objective ethical evaluation must be invariant under locally varying transformations of agents' evaluative frames, then a connection field is necessitated. The derivation establishes the connection's existence and constrains its transformation properties, but it does not, on its own, reveal the connection's *internal ethical structure*, its relationship to the three fundamental ethical dimensions.

The present paper's characterization of the ethical matter fields supplies the missing piece. The connection field must couple to *all three* ethical matter fields: it must organize axiological content ($\mathcal{G}$), deontic content ($\mathcal{R}$), *and* aretaic content ($\mathcal{V}$) into frame-independent evaluations. But a connection can couple to a matter field only if it has the internal structure to act on that field—to transform its components, to enforce coherence in its frame-dependent representation, to transport its content across locally varying evaluative frames.

The structural point can be stated precisely by drawing on the gauge-theoretic apparatus. In gauge theory, the connection takes values in the Lie algebra of the gauge group, and the Lie algebra's generators determine which matter fields the connection can couple to and how. A connection whose Lie algebra lacks generators corresponding to a given type of charge simply cannot couple to matter carrying that charge; the coupling is structurally impossible.[38]

The ethical analog is this: the connection field (good will) must have internal structure reaching into all three ethical dimensions (an axiological aspect, a deontic aspect, and an

[38] In quantum chromodynamics, for instance, the gluon field (the gauge connection of the strong force) takes values in the Lie algebra $\mathfrak{su}(3)$, which has eight generators corresponding to eight "color charges." The gluon field can couple to quarks (which carry color charge) but not to electrons (which do not). The coupling structure is determined by the Lie algebra, not by an external stipulation.

aretaic aspect) precisely *because* it must couple to $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ alike. A connection that were purely deontic (responsive only to the structure of obligation, with no internal axiological or aretaic structure) could not organize axiological content into frame-independent evaluations. It could enforce coherence in agents' representations of duty but would have no purchase on their representations of the good or of virtue. Similarly, a purely aretaic connection could organize virtue-content across frames but could not enforce coherence in the deontic or axiological fields. Only a connection that is itself *constitutively interdimensional*, i.e., that has generators along all three ethical directions, can do the work that ethical local invariance requires of it.

This gives a structural explanation of why the good will, as Kant describes it, is simultaneously deontic, aretaic, and axiological. These are not three independent philosophical claims about the good will. They are three manifestations of a single structural necessity: the ethical connection field must be interdimensional because the ethical matter fields it organizes are three-dimensionally structured. The connection's deontic aspect (its orientation toward the moral law) is what enables it to couple to $\mathcal{R}$. Its aretaic aspect (its constitutive relationship to moral worth) is what enables it to couple to $\mathcal{V}$. Its axiological aspect (its unconditional value) is what enables it to couple to $\mathcal{G}$. Kant's tripartite characterization of the good will is the philosophical expression of a structural requirement that the tensor-field model makes formally visible.

Why the connection must be syntegral. The preceding argument establishes that the ethical connection field must be constitutively interdimensional: it must have axiological, deontic, and aretaic aspects in order to couple to all three ethical matter fields. But this leaves open the *character* of the connection's interdimensionality. A connection could, in principle, be interdimensional but internally antagonistic, i.e., its axiological and deontic aspects could pull in opposing directions, or its aretaic aspect could be in tension with the other two. The syntegrity analysis of §4.5.4 provides the formal vocabulary for asking: what is the sign structure of the connection field's internal couplings?

I argue that the good will has an essentially *syntegral* internal structure: its axiological, deontic, and aretaic aspects are not merely co-present but mutually reinforcing—positively coupled across all three dimensions. The argument draws on three convergent lines of reasoning.

First, the Groundwork's characterization. Consider what Kant says about the good will in its most abstract characterization. The good will is *good*, forit is the only thing good without limitation (axiological: positive diagonal). It acts *from duty*, for its constitutive relationship to the moral law is an orientation toward rightness (deontic: positive diagonal). And it *is* moral worth, for it is what makes an agent's character genuinely admirable (aretaic: positive diagonal). All three diagonal entries are positive. But more significantly, the three aspects are not merely co-present; they are *mutually constitutive*. The good will's goodness is not independent of its orientation toward duty: it is good *because* it is righteous. Its moral worth is not independent of its goodness: it has worth *because* its orientation toward the moral law is genuinely valuable. And its righteousness is not independent of its moral worth: it fulfills duty *from* the right kind of character, not merely in external conformity with it. The off-diagonal couplings are all positive: each aspect of the good will reinforces and is reinforced by the others. This is exactly the syntegrity condition applied to the internal

structure of the connection field.

Second, the Doctrine of Virtue's specification. When the good will is considered not in the abstract but as embodied in human moral life (as it must be if it is to serve as the connection field organizing the ethical matter fields of actual moral communities) Kant specifies that its exercise constitutively requires a *dynamical harmony of respect and love*. In the Doctrine of Virtue, Kant argues that moral duties of respect and love are "essentially always connected together with one another in one duty according to the law": love must be tempered by respect (so that benevolence does not humiliate the beneficiary), and respect must be enriched by love (so that non-arrogance does not degenerate into cold distance).[39] This dynamical harmony is syntegrity as embodied in interpersonal moral dynamics.

To see why, consider how the two moral forces map onto the three ethical dimensions. Respect is primarily deontic: it consists of being moved by the thought of duty to comply with duties that maintain moral boundaries and impenetrability. But it is simultaneously axiological (it recognizes the unconditional worth, the *dignity*, of persons) and aretaic (the capacity for respect is a fundamental moral disposition without which humans cannot be genuine moral subjects). The three aspects of respect are synergistic: respecting someone's dignity *is* fulfilling a duty *is* exercising a virtuous disposition. Love (practical benevolence) is primarily axiological: it consists of being moved by the thought of duty to promote another's morally permissible happiness and ends. But it is simultaneously deontic (it fulfills duties of beneficence) and aretaic (the capacity for practical love is a fundamental moral disposition). Again, the three aspects are synergistic: promoting another's genuine good *is* fulfilling a duty of love *is* exercising a virtuous disposition. Each moral force is itself internally syntegral, i.e., positively coupled across all three ethical dimensions.

And the *balancing* of respect and love is itself a syntegral inter-force coupling. Each moral force makes the other *better*, not worse: respect prevents love from becoming paternalistic or boundary-violating; love prevents respect from becoming cold isolation. The coupling between the two forces is positive and mutually reinforcing: each sustains and enhances the other. This is the inter-field coupling with positive sign that the syntegrity condition requires.

Third, the moral-balancing argument as a syntegrity necessity proof. In other work, I argue that the dynamical harmony of respect and love is not an optional embellishment of moral life but a *condition for the possibility of moral community*.[40] Respect without love produces total moral dispersion (the purely respectful egoists of the moral-balancing argument); love without respect produces total moral collapse (the purely selfless lovers). Only their syntegral coupling, their mutual, positive, reinforcing sustains a stable moral configuration. The moral-balancing argument is, in the present framework, a *proof that the connection field must be syntegral*: an antisyntegral or partially syntegral connection cannot

[39] Kant, *Doctrine of Virtue*, 6:448–49. See Sanchez Borboa (2023) for a systematic account of this passage and the moral-balancing argument it contains.

[40] Sanchez Borboa (2023). The argument shows that purely respectful humans (respectful egoists) face a problem of total moral dispersion—they cannot be moved by the thought of duty to come morally closer by adopting each other's ends—while purely loving humans (selfless lovers) face a problem of total moral collapse—they cannot be moved by the thought of duty to maintain their moral boundaries and the separateness of persons. In either case, genuine moral interaction between humans is impossible. Only the joint exercise of respect and love, i.e., their dynamical harmony makes moral community possible.

sustain moral community, and moral community is the condition under which ethical local invariance (the requirement from which the connection field was derived in the first place) is exercised by embodied human agents.

The convergence of these three lines of reasoning yields a structural result: the good will's syntegral character is not one possible configuration among many that the connection field might happen to have. It is the *only* configuration that satisfies the structural requirements for moral community among embodied human agents. The Geometry paper derives the connection field from the requirement of ethical local invariance. The present paper's syntegrity analysis identifies the formal ideal toward which positively coupled ethical dynamics converge. The Balancing Forces argument shows that this ideal—the dynamical harmony of respect and love—is a condition for the possibility of the moral community in which ethical local invariance is exercised. Together, they establish that the connection field must be interdimensional (from the matter-field analysis), that its interdimensional structure must be syntegral (from the moral-balancing argument), and that its syntegral structure is the unique configuration that sustains the moral community on which objective ethical evaluation depends.

The good will, on this analysis, does not merely *satisfy* the syntegrity condition (as though syntegrity were an external standard against which the good will happens to measure up). The good will *essentially embodies* syntegrity: its constitutive structure *is* the dynamical harmony of axiological goodness, deontic rightness, and aretaic virtue, mutually reinforcing across all dimensions. The formal ideal derived from the structure of the ethical matter fields (the syntegrity condition on $\mathscr{E}_{\mathcal{H}}$) turns out to be realized in the structure of the ethical form (the connection field). The matter reveals the ideal; the form embodies it.[41]

The internal algebra of the connection. The interdimensionality of the connection has further consequences. If the connection field has components along the axiological, deontic, and aretaic directions, these components stand in algebraic relations to one another. In the language of gauge theory: the connection takes values in a Lie algebra whose generators include axiological, deontic, and aretaic generators, and the commutation relations among these generators determine the coupling structure of the theory.

A natural question arises: do the axiological, deontic, and aretaic components of the connection commute with one another?

If they commuted, i.e., if the order in which one applies duty-oriented and value-oriented considerations were irrelevant, then the connection's internal Lie algebra would be abelian. But there is strong reason to think that the ethical connection's internal algebra is *nonabelian*. The order in which one applies the connection's different dimensional aspects matters. Orienting first toward the moral law and then asking "what is good within the constraints of

[41] This result also illuminates why Kant treats the Stoic sage and the holy will as regulative ideals. They are ideals of *complete syntegrity*, i.e., agents whose good will is fully syntegrally expressed across the full range of ethically relevant cases, whose respect and love are in perfect dynamical harmony in every moral interaction. The moral-balancing argument shows that this ideal is not merely aspirational but structurally necessary: the syntegral configuration is what moral community *requires*, even if no finite human agent fully instantiates it. The distance of an actual agent's good will from full syntegral expression is, on this analysis, a formally precise measure of the distance between their moral character and the ideal of the sage—the "measuring stick of moral progress" that Kant describes in the *Critique of Practical Reason* (5:32–33) and that the syntegrity concept now gives tensorial articulation.

duty" yields a different evaluative landscape than orienting first toward the good and then asking "which duties arise from the structure of value." This is not a merely practical observation about how agents happen to deliberate; it is a structural feature of the ethical domain. It is, in fact, the substantive content of the longstanding disagreement between Kantian and consequentialist traditions—a disagreement that, in the present framework, amounts to a disagreement about the ordering of the connection's internal generators.[42]

The nonabelian character of the connection's internal algebra has a further consequence that connects to the present paper's central apparatus. In a nonabelian gauge theory, the connection's curvature (field strength) is richer than in the abelian case: it includes "self-interaction" terms arising from the noncommutativity of the Lie algebra generators.[43] If the ethical connection's internal algebra is nonabelian, then its curvature—the gauge-invariant content that the Geometry paper identifies as "the light by which the good will shines"—includes cross-dimensional terms that arise from the noncommutativity of the axiological, deontic, and aretaic generators.

Cross-dimensional curvature and the objectivity of coupling. This brings us to what I take to be the deepest structural insight that emerges from bringing the companion papers and the present paper together.

The Geometry paper's curvature (field strength) encodes the objective, frame-independent ethical content that the connection carries. If the connection has a three-dimensionally structured internal space (with axiological, deontic, and aretaic generators), then the curvature itself has components *along and across* these dimensions. It is not a single number or a simple object; it is structured by the same three-dimensional architecture that structures the matter fields:

- The *intra-dimensional* curvature components (axiological-axiological, deontic-deontic, aretaic-aretaic) encode the frame-independent ethical content *within* each fundamental dimension: the objective structure of goods, the objective structure of duties, the objective structure of virtuous character.

- The *cross-dimensional* curvature components (e.g., axiological-deontic, deontic-aretaic, aretaic-axiological) encode the frame-independent content of the coupling *between* dimensions: the objective structure of how goods, duties, and virtues bear on one another.

[42] This reframing has a clarifying consequence. If the axiological and deontic components of the connection commuted, the Kantian-consequentialist disagreement would dissolve: the order of application would be irrelevant, and both traditions would converge on the same evaluative structure. The persistence of the disagreement across centuries of sustained philosophical scrutiny is evidence that the components do *not* commute—that the connection's internal Lie algebra is nonabelian. This connects to the Geometry paper's nonabelian gauge group example (the vocabulary-transformation group, where the order of translation between Confucian, Kantian, and Aristotelian frameworks matters). The noncommutativity at the level of evaluative frames, which the Geometry paper establishes, is now reflected at the level of the connection field's own internal structure.

[43] In an abelian gauge theory (like electromagnetism with gauge group $U(1)$), the field strength is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$: a simple difference of derivatives of the connection. In a nonabelian gauge theory (like quantum chromodynamics with gauge group $SU(3)$), the field strength is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$: the abelian part plus a commutator term that encodes the self-interaction of the gauge field. The commutator term is absent in the abelian case because abelian generators commute.

The cross-dimensional curvature components are where the present paper's inter-field coupling (§3.3) meets the Geometry paper's gauge-invariant content. The coupling operators $C^{\mathcal{A}\to\mathcal{B}}$ (Definition 4) describe how the three matter fields influence one another; they are, in general, frame-dependent, i.e., their component representation varies across evaluative vocabularies. But the cross-dimensional curvature of the connection extracts the *gauge-invariant core* of that coupling: the part of the inter-field influence that survives every local re-framing and constitutes the objective ethical structure of the interdependence among the good, the right, and moral worth.

This yields a precise and, I believe, novel account of what makes the coupling among ethical dimensions *objective*. The coupling between $\mathcal{G}$ and $\mathcal{R}$ is not objective merely because both fields exist and influence each other; their mutual influence could, in principle, be an artifact of a particular evaluative framing. The coupling is objective because the connection field, which enforces frame-independence, has cross-dimensional curvature that registers the coupling as gauge-invariant content. The cross-dimensional curvature is the "seal of objectivity" on the inter-field coupling: it certifies that the coupling is a feature of the ethical reality, not a feature of any particular evaluative vocabulary.[44]

The circle closes. The picture that emerges is one in which the form and the matter of the ethical domain are not merely complementary but *mutually illuminating*. The matter fields ($\mathcal{G}$, $\mathcal{R}$, $\mathcal{V}$) reveal that the connection field must be internally interdimensional, because only an interdimensional connection can couple to all three fields. The connection field, through its cross-dimensional curvature, certifies which features of the inter-field coupling are objectively real (gauge-invariant) and which are artifacts of evaluative framing. The tripartite structure of the matter determines the internal structure of the form; the form, in turn, determines which features of the matter's coupling structure are objective.

This mutual illumination could not have been seen from either paper alone. The Geometry paper, working at the level of formal requirements on objective evaluation, derives a connection field with determinate transformation properties and gauge-invariant curvature, but does not reveal the connection's internal ethical dimensionality. The present paper, working at the level of the substantive ethical content, characterizes the matter fields and their couplings, but does not establish which features of the coupling are frame-independent. Only when the form and the matter are brought together does the full structure become visible: an ethically interdimensional connection field whose cross-dimensional curvature encodes the objective structure of the coupling among the good, the right, and moral worth.

The full formal development of this interaction (the construction of the connection's internal Lie algebra, the derivation of its curvature components along and across the ethical dimensions, and the precise characterization of how the cross-dimensional curvature determines the objective content of the inter-field coupling) is a substantial undertaking that lies beyond the scope of the present paper. But the structural insight is, I believe, already clear: the matter illuminates the form, the form certifies the matter, and the integration of the two

[44] This account connects to the Geometry paper's characterization of the good will's "light" as curvature/field strength. The good will shines by its own light because its curvature—the full curvature, including the cross-dimensional components—is gauge-invariant. Part of what that light reveals is the objective structure of the coupling among the ethical dimensions. The good will does not merely enforce coherence within each dimension separately; it enforces coherence across dimensions, and the cross-dimensional curvature is the objective content of that cross-dimensional coherence.

yields a picture of the ethical domain that is richer than either alone could provide.[45]

4.9 Implications for applied ethics and moral practice

The tensor-field model has diagnostic power that extends beyond theoretical moral philosophy. When faced with a complex ethical situation (whether in institutional design, policy evaluation, professional ethics, or personal moral development) the framework suggests a systematic approach: examine not only the evaluative content along each dimension (the diagonal entries) but the coupling structure (the off-diagonal entries and the inter-field coupling blocks).

Consider institutional design. A well-designed institution, on this model, is one whose structure supports positive cross-dimensional coupling: an institution in which doing the right thing also tends to produce good outcomes and to cultivate virtuous dispositions in its members. The off-diagonal entries of $\mathscr{E}^{\mathcal{H}}(w)$, evaluated at the institution's characteristic ethical situations, should be predominantly positive. A poorly designed institution is one that allows or encourages antagonistic coupling: an incentive structure that rewards good outcomes ($\mathcal{G}$ positive) through mechanisms that corrode integrity ($C^{\mathcal{G}\to\mathcal{V}} < 0$) or that encourage cutting ethical corners ($C^{\mathcal{G}\to\mathcal{R}} < 0$). The tensor-field framework makes the diagnosis structurally precise: the problem is not that the institution fails on any single dimension but that its coupling structure is pathological.

This diagnostic applies at both levels of the tensor structure. At the intra-dimensional level, one can ask whether an institution's promotion of welfare comes at the cost of autonomy (negative off-diagonal in $\mathcal{G}$), or whether its enforcement of justice comes at the cost of fidelity (negative off-diagonal in $\mathcal{R}$). At the inter-dimensional level, one can ask whether the institution's axiological success (good outcomes) is coupled positively or negatively to its deontic profile (right conduct) and its aretaic profile (virtuous culture).

The framework is equally applicable to personal moral development. An agent who cultivates individual virtues in isolation (attending only to the diagonal entries of $\mathcal{V}$) may miss the coupling structure that determines whether those virtues reinforce or undermine one another. The classical Aristotelian insight that practical wisdom is the "architectonic" virtue, i.e., the virtue that organizes all others, can be given a tensor-field reading: practical wisdom is the virtue whose positive coupling to all other virtues (large positive off-diagonal entries $\mathcal{V}_{4j}$ for all j) ensures that the aretaic tensor as a whole is well-configured.[46]

[45] This program of integrating the connection and the matter fields into a unified dynamical theory is, in the electromagnetic analogy, the program of coupled Maxwell-Lorentz theory: the gauge field and the matter fields co-evolve according to coupled equations of motion. The ethical analog would be a theory in which the connection (good will) and the matter fields ($\mathcal{G}$, $\mathcal{R}$, $\mathcal{V}$) co-evolve according to coupled ethical dynamics, each shaping the other. This is a natural and, I believe, promising direction for future work in the research program.

[46] The framework also has implications for the ethics of artificial intelligence and socio-technical systems, connecting to the applied dimensions of the broader research program. An AI system or a socio-technical institution can be understood as a participant in the ethical field whose design and behavior affect the coupling structure of the ethical landscape. A system that optimizes for a narrow measure of the good (a single diagonal entry of $\mathcal{G}$) while ignoring or degrading the coupling to the right and to virtue is, on this model, a system that risks initiating vicious spirals by disrupting the positive inter-field coupling that well-functioning moral communities depend on. This connects the formal framework to practical questions

The upshots developed in this section are consequences of the tensor-field model's core structural features: the multidimensionality of the ethical fields, the off-diagonal coupling terms, and the dynamic inter-field coupling. They are not available from within any single ethical tradition, nor from informal accounts of ethical interdependence that lack the tensor structure. The framework does not replace substantive ethical inquiry—it does not tell us *which* goods are genuine, *which* duties are binding, or *which* character traits are virtuous—but it provides a formal architecture within which those substantive questions can be posed, related, and investigated with a precision not previously available. The next section examines where the analogy with physics breaks down and argues that the breakdowns are, as in the companion papers, philosophically productive rather than problematic.

5 Disanalogies and Their Productivity

The electromagnetic analogy has done considerable work in motivating and structuring the tensor-field model. But the analogy is imperfect, and it would be a defect of the paper to leave the imperfections unexamined. This section identifies five principal disanalogies between the electromagnetic and ethical cases. Following the strategy of the companion paper's treatment of disanalogies between gauge theory and moral philosophy, I argue that each disanalogy is philosophically productive: it marks a place where the ethical domain has distinctive structure that the physical analogy does not capture but does help us see.[47]

5.1 The normative character of the ethical field

The electromagnetic field describes what is the case: it is a component of the physical world whose configuration is a matter of empirical fact. The electric and magnetic field values at a point in spacetime are determined by the distribution of charges and currents and by the prior history of the field. The field is descriptive: it reports, it does not prescribe.

The ethical tensor fields $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ are different. They encode not merely what agents happen to value, what obligations happen to be recognized, or what character traits happen to be prevalent, but what is genuinely good, genuinely right, and genuinely virtuous—or at least what a framework committed to the objectivity-as-invariance principle is committed to treating as such. The ethical field prescribes: it specifies not what evaluative relations obtain as a matter of sociological fact but what evaluative relations ought to obtain if ethical evaluation is to be objective.

This is a genuine difference, and it would be a mistake to minimize it. But it is accommodated by the conditional structure of the argument, as in the companion papers. The derivation of the tensor-field model is conditional: *if* one accepts that ethical evaluation has three irreducible and interdependent dimensions, *and if* one accepts that the coupling

about the design, governance, and evaluation of powerful socio-technical systems.

[47] The Geometry paper (Sanchez Borboa, manuscript B, §4) examines four disanalogies between gauge theory and moral philosophy: the non-uniqueness of the ethical connection, the self-referentiality of the ethical gauge field, the absence of a variational principle, and the normative character of the connection. The disanalogies examined here are distinct from (though in some cases structurally related to) those examined there, since the present paper concerns the "matter" side of the framework (the ethical tensor fields) rather than the "form" side (the connection/gauge field).

among these dimensions is dynamic and constitutive rather than accidental, *then* the rank-2 tensor field is the appropriate formal representation. The argument does not assert that the ethical field exists as a brute metaphysical fact in the way the electromagnetic field does. It shows that the tensor-field structure is the form that the ethical landscape must take if the premises are accepted. A moral realist can read the ethical fields as objective features of moral reality; a constructivist can read them as structures constituted by rational agents' evaluative activity; an anti-realist can read the argument as clarifying what the structure of the ethical domain would have to be if ethical objectivity were achievable.[48]

5.2 The absence of precise quantitative coupling constants

In electromagnetism, the coupling between the electric and magnetic fields is governed by precise, empirically determined constants: the speed of light c, the permittivity ε_0 and permeability μ_0 of free space. Maxwell's equations specify not merely that the fields are coupled but *exactly how strongly* and with *exactly what mathematical form*. This quantitative precision is what makes electromagnetism a predictive theory: given initial conditions and boundary conditions, the field equations determine the future evolution of the fields with arbitrary accuracy.

The ethical coupling principles identified in §3.6 have no such quantitative precision. The axiological-deontic coupling principle states that changes in $\mathcal{G}$ generate changes in $\mathcal{R}$, and vice versa, but it does not specify the "coupling constant" that governs the strength of this influence, nor does it specify the precise mathematical form of the coupling operator $C^{\mathcal{G}\to\mathcal{R}}$. The coupling equations (4–6) use the symbol $\sim$ (structural proportionality) rather than $=$ (exact equality) deliberately: they are structural constraints, not quantitative laws.

This is a genuine disanalogy, but it is not a defect. It reflects the nature of the ethical domain. There are at least two reasons to think that the absence of precise coupling constants is appropriate rather than problematic.

First, the ethical coupling is plausibly *situation-dependent* in a way that the electromagnetic coupling is not. The strength with which changes in the axiological landscape generate deontic consequences depends on the character of the changes, the structure of the existing deontic landscape, and the features of the moral community in which the coupling operates. There may be no single constant that governs the $\mathcal{G} \to \mathcal{R}$ coupling across all ethical situations, just as there is no single constant that governs the relationship between economic conditions and political institutions across all societies. The coupling operators are field-valued (they vary across W), and their determination is a task for substantive moral inquiry, not for formal framework-building alone.

Second, even without precise quantitative constants, the tensor-field framework provides *structural* information that is philosophically valuable and that purely verbal accounts cannot match. It tells us that the coupling exists, that it is bidirectional but asymmetric (§3.4), that it can be positive (synergistic) or negative (antagonistic), that its sign and magnitude vary across the ethical landscape, and that the pattern of coupling generates emergent dynamics (virtuous and vicious spirals) not capturable by any single dimension. This is more than what

[48]This metaethical ecumenism parallels the companion paper's treatment of the normative character of the ethical connection (Sanchez Borboa, manuscript B, §4.4). The formal framework is compatible with a range of metaethical positions; what it provides is structural articulation, not metaphysical commitment.

was available before the framework, even if it is less than what electromagnetism provides. The appropriate comparison is not with the precision of Maxwell's equations but with the state of moral theory without the tensor-field model.[49]

5.3 Agent-dependence of the ethical field

The electromagnetic field exists independently of the charged particles whose interactions it mediates. The field is "out there," pervading space, with its own degrees of freedom and its own dynamical evolution. Charges respond to the field and generate the field, but the field is not constituted by the charges: it has an independent ontological status.

The ethical tensor fields, by contrast, are at least partly constituted by the agents whose evaluative activity they characterize. The axiological landscape (what is good and bad) is not a feature of the world that exists entirely independently of beings capable of valuing; the deontic landscape (what is right and wrong) does not exist independently of agents capable of recognizing and responding to obligations; the aretaic landscape (what is virtuous and vicious) does not exist independently of agents who have character.[50] The ethical field is not "out there" awaiting discovery; it is, at least in part, enacted, sustained, and reshaped by the agents whose moral life it characterizes.

This disanalogy echoes the self-referentiality that the Geometry paper identifies for the ethical connection field, now extended to the matter fields. And it is similarly productive. It captures a distinctive feature of the ethical domain: the landscape of goods, duties, and virtues is not fixed independently of moral practice. When agents cultivate new virtues, the aretaic field $\mathcal{V}$ genuinely changes, not merely in its representation but in its content. When a community recognizes new forms of harm, the axiological field $\mathcal{G}$ genuinely expands. The agent-dependence of the ethical field is what makes moral progress (and moral regress) possible: the fields can be improved or degraded by the very agents whose evaluative activity they characterize.

This has a further consequence that connects to the spiraling dynamics of §4.5. In electromagnetism, the field and the charges evolve according to coupled equations, but the charges cannot "refuse" to respond to the field: a charged particle in an electric field accelerates; it has no choice. In ethics, agents can fail to respond to the ethical field, i.e., they can ignore the good, violate the right, neglect the cultivation of virtue. This means that the coupling between ethical agents and the ethical field is *voluntary* in a way that the coupling between charges and the electromagnetic field is not. The possibility of moral failure (and of moral

[49] As an instructive parallel, Faraday's field concept was enormously productive long before Maxwell supplied the precise equations. The idea that electric and magnetic influences propagate through a field pervading space, rather than acting instantaneously at a distance, restructured physical understanding even before the quantitative laws governing the field were known. The tensor-field model of the ethical domain may stand in a similar relation to its own eventual quantitative development: the structural insight (that the ethical dimensions are dynamically coupled tensor fields) may prove productive even before precise coupling equations are available.

[50] This claim is compatible with a range of positions on the mind-dependence of moral facts. A robust moral realist may hold that the ethical fields supervene on mind-independent moral facts but are *epistemically accessible* only through agents' evaluative activity. A constructivist will hold that the ethical fields are constituted by that activity. The present point is that, on either view, the fields are bound up with agency in a way that the electromagnetic field is not bound up with charged matter.

achievement) is built into the structure of the ethical domain in a way that has no electromagnetic analog. The framework does not explain *why* agents sometimes fail to respond to the ethical field (that is a question for moral psychology), but it provides a structural characterization of what such failure amounts to: a disruption of the coupling that, in a well-functioning moral community, sustains the positive inter-field dynamics.[51]

5.4 The dimensionality question

In electromagnetism, the two-field structure (electric and magnetic) is not a contingent taxonomic choice. It is forced by the mathematical structure of spacetime: given four-dimensional Minkowski spacetime and the Lorentz group, the antisymmetric rank-2 tensor has exactly six independent components, which decompose naturally into three electric and three magnetic components. The two-field structure is a theorem, not a hypothesis.

In ethics, the three-dimensional structure (axiological, deontic, aretaic) is a widely accepted but not universally agreed-upon taxonomy. Why three fundamental dimensions and not two, or four, or more? The question is legitimate and should be addressed directly.

The three-dimensional structure has strong independent motivation. The tripartite distinction between the good, the right, and moral worth goes back at least to Rawls's characterization of moral theory and has deep roots in the structure of moral philosophy itself: the persistence of three irreducible traditions (consequentialism, deontology, virtue ethics) across centuries of sustained critical scrutiny is itself evidence that the three dimensions capture something real about the structure of ethical evaluation. The irreducibility arguments of §1.5.1 provide further support: if any of the three dimensions could be reduced to the others, one would expect the reducing tradition to have achieved consensus by now, or at least to have made the case for reduction compelling enough to dissolve the rival traditions. This has not occurred, and the best explanation is that the three dimensions are genuinely irreducible.

Nevertheless, the tensor-field framework is *modular* with respect to dimensionality. If a compelling case were made for a fourth fundamental ethical dimension (for instance, an aesthetic dimension that is irreducible to the axiological, deontic, and aretaic), the framework could accommodate it by expanding the block tensor $\mathscr{E}^{\mathcal{H}}(w)$ to a 4×4 block structure. Conversely, if it were shown that two of the three dimensions can be unified (as some philosophers have argued for the good and virtue, or for the right and virtue), the framework could be simplified accordingly. The dynamic-coupling architecture does not depend on the number three; it depends on there being *multiple irreducible and interdependent dimensions*, whatever their number. The specific dimensionality is an open empirical-normative question that the framework parametrizes rather than settles.[52]

[51] This connects to the Geometry paper's characterization of moral failure as gauge-structural defect (Sanchez Borboa, manuscript B, §4.4): an agent who fails to act from the good will disrupts the connection, degrading the community's capacity for objective ethical evaluation. The present paper adds that such failure also disrupts the coupling among the ethical matter fields, potentially initiating the vicious spirals described in §4.5.2.

[52] The modularity of the framework extends to the intra-dimensional level as well. The number of sub-dimensions within each field ($n_{\mathcal{G}}$, $n_{\mathcal{R}}$, $n_{\mathcal{V}}$) is not fixed by the framework. Different ethical traditions, or different applications, may identify different sub-dimensions as fundamental. The tensor-field model

5.5 Symmetric versus general tensors

The Faraday tensor is antisymmetric: $F_{\mu\nu} = -F_{\nu\mu}$. This antisymmetry is not a convention or an approximation; it is forced by the structure of the Lorentz group and the requirement that the tensor encode a genuine two-form on spacetime. It is a deep feature of the electromagnetic field, one that constrains the possible field configurations and determines the mathematical structure of the theory.

The ethical tensor, as argued in §3.4, is general: $\mathcal{F}_{ij}(w) \neq \mathcal{F}_{ji}(w)$ in general, and no symmetry group forces the tensor to be symmetric, antisymmetric, or to have any particular index symmetry. The ethical domain has more coupling degrees of freedom than the electromagnetic domain: where the Faraday tensor's antisymmetry reduces its independent components from 16 to 6 (in four dimensions), the generality of the ethical tensor leaves all n^2 components independent (for each $n \times n$ intra-dimensional tensor).

This disanalogy might seem to weaken the analogy: if the ethical tensor lacks the elegant symmetry properties of its electromagnetic counterpart, in what sense is the analogy illuminating?

The answer is that the *greater generality* of the ethical tensor is itself one of the analogy's most productive results. It reveals that the ethical domain has a richer coupling structure than the electromagnetic domain, i.e., that the relationships among ethical dimensions are more complex, more directional, and less constrained by formal symmetry than the relationships among components of the electromagnetic field. This is not a failure of the formalism; it is a discovery about the subject matter.

Moreover, the generality opens a research program that the antisymmetric case forecloses. In electromagnetism, the symmetry properties of $F_{\mu\nu}$ are known and fixed; there is nothing further to investigate about the tensor's index structure. In ethics, the symmetry properties of $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ are *open questions*: for which sub-dimension pairs is the coupling approximately symmetric? For which is it strongly asymmetric? Are there regions of the ethical landscape where the tensor is approximately antisymmetric, suggesting a structural parallel with electromagnetism that is deeper than the general case? These are substantive questions about the structure of ethical reality that the framework makes formally precise and, at least in principle, investigable.[53]

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accommodates any such choice and provides the formal structure for representing whatever coupling structure obtains among the chosen sub-dimensions. This flexibility is a feature, not a bug: it allows the framework to serve as a common formal language for traditions that disagree about the internal structure of the ethical dimensions.

[53] One might decompose any general rank-2 tensor into its symmetric and antisymmetric parts: $\mathcal{F}_{ij} = \frac{1}{2}(\mathcal{F}_{ij} + \mathcal{F}_{ji}) + \frac{1}{2}(\mathcal{F}_{ij} - \mathcal{F}_{ji})$. The symmetric part encodes the "mutual" coupling (the extent to which the i-to-j and j-to-i influences agree), and the antisymmetric part encodes the "directional" coupling (the extent to which they disagree). This decomposition provides a formal tool for characterizing the directionality of ethical coupling in any given situation. A situation in which the antisymmetric part dominates is one in which the coupling is strongly directional; a situation in which the symmetric part dominates is one in which the coupling is approximately mutual. The ethical significance of these structural possibilities (and the question of which obtains in which regions of the ethical landscape) is a natural direction for future investigation.

Summary

The five disanalogies examined in this section (the normative character of the ethical field, the absence of precise coupling constants, the agent-dependence of the field, the dimensionality question, and the generality of the ethical tensor) are not defects in the tensor-field model. They are structural features of the ethical domain that the model, precisely by pressing the electromagnetic analogy to its limits, makes visible and tractable. Each disanalogy opens a research direction: the metaethical interpretation of the ethical fields, the search for more precise coupling principles, the dynamics of agent-dependent fields, the determination of the ethical domain's fundamental dimensionality, and the investigation of the ethical tensor's symmetry properties. That the analogy with electromagnetism is imperfect is, as in the companion papers, precisely what makes it productive: the points of disanalogy mark the places where the ethical domain has structure that electromagnetism does not, and the tensor-field vocabulary provides new tools for exploring that structure.

6 Conclusion

This paper has argued that the fundamental dimensions of ethical evaluation—the axiological ($\mathcal{G}$), the deontic ($\mathcal{R}$), and the aretaic ($\mathcal{V}$)—are fruitfully modeled as dynamically coupled rank-2 tensor fields, structurally analogous to (but importantly disanalogous from) the dynamically coupled electric and magnetic fields of electromagnetism.

The argument has moved through five stages. Section 1 established the philosophical groundwork: the three dimensions are both internally multidimensional and mutually interdependent in ways that resist reduction, and the pattern of their interdependence calls for a formal framework that represents coupling structure, not merely dimensional magnitudes. Section 2 developed the electromagnetic analogy, identifying the structural parallels—non-reductive unification, dynamic coupling, emergent phenomena, the field concept, and the tensor representation—that motivate the formal proposal. Section 3 presented the core formalism: each dimension is modeled as a rank-2 tensor field over its internal sub-dimensions, the three fields are dynamically coupled by inter-field coupling operators, and the complete ethical field configuration at any case w is encoded in a holistic block tensor $\mathscr{E}^{\mathcal{H}}(w)$ that unifies the three dimensions without reducing any to the others. Section 4 drew out the upshots: a non-reductive unification of the three ethical traditions, a formal model of the ethical landscape, a structural characterization of ethical dilemmas (as negative off-diagonal coupling) and moral synergies (as positive off-diagonal coupling), the emergence of non-additive virtuous and vicious spirals from the inter-field coupling, a reframing of classical moral disagreements as structural hypotheses about coupling magnitudes, a connection to the companion papers' gauge-theoretic foundations, and diagnostic implications for applied ethics and institutional design. Section 5 examined the principal disanalogies with electromagnetism and argued that each is philosophically productive rather than problematic.

Two remarks on the paper's place in the broader research program are in order.

Form and matter. The companion papers characterize the *form* of the ethical landscape: the Symmetry paper establishes that objectivity-as-invariance bootstraps from minimal coherence to robust impartiality; the Geometry paper shows that the demand for lo-

cal invariance necessitates a connection field (identified with the Kantian good will) whose curvature encodes gauge-invariant ethical content. The present paper characterizes the *matter*: the substantive ethical content—the landscape of goods, duties, and virtues and their interrelations—that the connection field organizes into frame-independent evaluations. Together, the three papers provide a formal framework in which both the form and the substance of the ethical domain are specified. The connection (good will) enforces coherence across locally varying evaluative frames; the tensor fields ($\mathcal{G}$, $\mathcal{R}$, $\mathcal{V}$) provide the structured content on which the connection acts; and the coupling among the tensor fields generates the dynamic ethical landscape whose frame-independent content the connection's curvature extracts.

The Maxwell parallel, extended. The introduction observed that Maxwell's unification of electricity and magnetism resolved a pre-existing impasse: the two phenomena were known to be connected, but the structure of the connection was unclear, and each attempt to reduce one to the other proved inadequate. The resolution was non-reductive dynamic coupling: neither field is fundamental; both are co-equal aspects of a single electromagnetic field governed by mutual coupling equations. The present paper has argued that the three fundamental ethical dimensions stand in an analogous impasse, and that the resolution takes an analogous form: non-reductive dynamic coupling among three co-equal tensor fields.

But Maxwell's unification was not the end of the story. It was the beginning of a research program that led, through special relativity, to the Faraday tensor's frame-independent formulation; through quantum mechanics, to quantum electrodynamics; and through the generalization of the gauge principle, to the Standard Model of particle physics, which unifies the electromagnetic, weak, and strong forces within a single gauge-theoretic framework. Each step deepened the understanding of the coupling structure that Maxwell had identified, and each generated theoretical and empirical results that could not have been anticipated from the unification alone.

The tensor-field unification of the axiological, deontic, and aretaic dimensions is, I suggest, at an analogous early stage. The present paper identifies the coupling structure and develops its immediate consequences. But the framework opens research directions whose full development lies ahead.

Among the most salient:

Ethical coupling equations. The coupling principles of §3.6 are structural constraints; they do not specify the precise form of the coupling operators $C^{\mathcal{A}\to\mathcal{B}}$. Developing more specific coupling equations, i.e., structural laws governing how changes in one ethical field generate changes in the others, would bring the ethical framework closer to the predictive power of Maxwell's equations. Whether such equations can be formulated with genuine precision, and what form they would take, is an open question.[54]

Source theory. In electromagnetism, the fields are generated by sources: charges and currents. The Schwinger source-theoretic approach to quantum field theory characterizes

[54] One natural approach would be to investigate whether the coupling operators satisfy conservation laws or variational principles analogous to those governing the electromagnetic coupling. The companion paper's discussion of ethical action principles (Sanchez Borboa, manuscript B, §4.3) suggests several candidates: a flatness principle (minimize cross-field tension), a richness principle (maximize the irreducible cross-field content), or a reflexive-equilibrium principle (select the coupling configuration that is self-consistent). Adapting these to the tensor-field context is a natural next step.

fields operationally, in terms of how they respond to sources and how sources respond to them. An ethical analog of source theory would characterize the ethical fields in terms of how they are generated and sustained by agents, institutions, and practices, i.e., the "sources" of the ethical field.[55]

Ethosystem dynamics. The tensor-field model as developed in this paper treats the domain W of ethically relevant cases as a fixed background over which the ethical fields $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ are defined. But the ethical domain is not a fixed stage on which ethical phenomena merely play out. Agents, institutions, ecosystems, and political structures constitute a dynamic psycho-bio-socio-eco-geo-political medium that both shapes and is shaped by the ethical fields it sustains. The ethical fields do not merely propagate through this medium; they dynamically couple to it, and it to them.

The structural analogy here is with general relativity. In special relativity, physical fields propagate on a fixed spacetime background (Minkowski spacetime). In general relativity, Einstein recognized that this background is itself dynamical: matter and spacetime co-determine each other through the Einstein field equations, which couple the stress-energy tensor (encoding the distribution of matter and energy) to the metric tensor (encoding the geometry of spacetime). Spacetime is not a container for physics; it is a participant in the dynamics.

The ethical analog would be a framework in which the medium through which ethical fields propagate, the *ethosystem*, is itself a dynamical structure that co-evolves with the ethical fields it sustains. On this view, a fuller model of the ethical case-space would replace the abstract case-coordinate w with a richer coordinate structure

$$\xi^M = (x^0,\ x^1, x^2, x^3,\ w^4, w^5, w^6, w^7, w^8, w^9)$$

comprising ten dimensions: four spatiotemporal ones and six properly ethosystemic ones ($M = 0, 1, \ldots, 9$). The richer structure of this ethosystem as a medium for ethics serves as a natural opportunity to extend the tensor-field approach to modeling ethically interesting phenomena. In particular, the geometry of this manifold is encoded in an ethosystemic metric tensor $\mathfrak{es}_{MN}$ (the ethical analog of the spacetime metric $g_{\mu\nu}$ in general relativity) whose components determine the "distances," couplings, and curvature structure of the ethosystemic medium at each point. The spatiotemporal coordinates locate ethical phenomena in physical space and time:

- $x^0 = t$: the temporal coordinate;
- x^1, x^2, x^3: spatial coordinates.

The ethosystemic coordinates encode the configuration of the medium in which ethical fields are embedded:

- w^4: the *psychological* dimension (cognitive capacities, affective dispositions, developmental stage);

[55] This connects to the applied side of the broader research program. Agents, institutions, and socio-technical systems can be understood as sources of the ethical field: they generate and sustain (or degrade and disrupt) the axiological, deontic, and aretaic content that the tensor-field model represents. A Schwinger-type source theory for the ethical field would provide formal tools for characterizing how different kinds of agents and institutions contribute to or detract from the ethical field configuration.

- w^5 : the *biological* dimension (embodiment, health, vulnerability, organismic need, and—since biology properly includes evolution, ecology, and behavior—ecological structure at the population, community, and ecosystem level);

- w^6 : the *social-cultural* dimension (including the multiplicative, intersectional structure of identity, i.e., race, gender, class, disability, etc. and their characteristically non-additive interactions);[56]

- w^7 : the *economic* dimension (production, distribution, exchange, labor, resource allocation, wealth, income, and capital);

- w^8 : the *geographic* dimension (place, climate, biosphere, natural resource distribution, and the physical-ecological setting broadly construed);

- w^9 : the *political* dimension (power, governance, representation, and collective decision-making).

The resulting ten-dimensional ethosystemic manifold has a $4 + 6$ structure: four extended spatiotemporal dimensions plus six ethosystemic dimensions whose configuration at each spatiotemporal point determines the effective ethical dynamics there. Just as general relativity models physical quantities as dynamically coupling to a dynamical spatiotemporal medium, ethosystem theory would model ethical fields as dynamically coupling to a dynamical ethosystemic medium, one whose own configuration is partly determined by the ethical fields it sustains, and whose configuration in turn shapes the ethical fields that propagate through it.[57]

The six ethosystemic dimensions are not independent: they couple to one another through off-diagonal terms in an ethosystemic metric, just as the ethical sub-dimensions within each field couple through the off-diagonal entries of the ethical tensors. The psychological and biological dimensions interact (embodiment shapes cognition and affect); the social-cultural and economic dimensions interact (economic structures shape and are shaped by cultural norms and intersectional identity); the geographic and political dimensions interact (resource distribution shapes governance, and governance shapes resource allocation). The ecological content that might seem to warrant a separate dimension is, on this analysis, encoded in the off-diagonal coupling between the biological and geographic dimensions (ecology being, at its core, the interaction between living systems and their physical setting) together with the couplings between these dimensions and the economic and social-cultural dimensions that

[56]This ethosystem model is thus rich enough to capture the complex dynamic structure of intersectional identity. This is crucial for recognizing that the ethical significance of occupying multiple marginalized positions is not the sum but the product of the significances of each marginalization.

[57]The dynamic co-determination of ethical fields and their ethosystemic medium is the ethical analog of what physicists call "backreaction": the way matter curves spacetime, which in turn affects matter's motion, which further curves spacetime, and so on. In the ethical case, backreaction takes the form of feedback between ethical field configurations and the psycho-bio-socio-eco-geo-political structures that sustain them: a community's ethical field configuration (its realized goods, fulfilled duties, cultivated virtues, and their couplings) reshapes the social, economic, ecological, geographic and political medium in which it is embedded, which in turn reshapes the conditions under which ethical fields evolve. Developing the formal apparatus for this co-determination, i.e., the ethical analog of the Einstein field equations, is a central task for future work.

mediate human ecological relations. The ten-dimensional model is the minimal structure that captures the irreducible dimensionality of the ethosystemic medium without introducing redundant coordinates, and the tensor architecture ensures that the coupling *between* dimensions is represented with the same formal precision as the content *along* them.[58]

The ethosystemic tensor $\mathfrak{es}_{MN}$ is a general (not necessarily symmetric) rank-2 tensor field on the ten-dimensional ethosystemic manifold. Like the ethical field tensors $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ developed in §3.4 (and for the same philosophical reason), the couplings between ethosystemic dimensions are directional, with the M-to-N influence structurally different from the N-to-M influence, i.e., the ethosystemic tensor is not constrained to be symmetric. We can represent this tensor as a matrix with the following structure:

$$\mathfrak{es}_{MN} = \left[\begin{array}{c|cc|cccccc} & 0\,(t) & 1{-}3\,(x^i) & 4\,(\psi) & 5\,(\beta) & 6\,(\sigma) & 7\,(\varepsilon) & 8\,(\gamma) & 9\,(\pi) \\ \hline 0\,(t) & \mathfrak{es}_{00} & \mathfrak{es}_{0i} & \mathfrak{es}_{0\psi} & \mathfrak{es}_{0\beta} & \mathfrak{es}_{0\sigma} & \mathfrak{es}_{0\varepsilon} & \mathfrak{es}_{0\gamma} & \mathfrak{es}_{0\pi} \\ 1{-}3\,(x^i) & \mathfrak{es}_{i0} & \mathfrak{es}_{ij} & \mathfrak{es}_{i\psi} & \mathfrak{es}_{i\beta} & \mathfrak{es}_{i\sigma} & \mathfrak{es}_{i\varepsilon} & \mathfrak{es}_{i\gamma} & \mathfrak{es}_{i\pi} \\ \hline 4\,(\psi) & \mathfrak{es}_{\psi 0} & \mathfrak{es}_{\psi i} & \mathfrak{es}_{\psi\psi} & \mathfrak{es}_{\psi\beta} & \mathfrak{es}_{\psi\sigma} & \mathfrak{es}_{\psi\varepsilon} & \mathfrak{es}_{\psi\gamma} & \mathfrak{es}_{\psi\pi} \\ 5\,(\beta) & \mathfrak{es}_{\beta 0} & \mathfrak{es}_{\beta i} & \mathfrak{es}_{\beta\psi} & \mathfrak{es}_{\beta\beta} & \mathfrak{es}_{\beta\sigma} & \mathfrak{es}_{\beta\varepsilon} & \mathfrak{es}_{\beta\gamma} & \mathfrak{es}_{\beta\pi} \\ 6\,(\sigma) & \mathfrak{es}_{\sigma 0} & \mathfrak{es}_{\sigma i} & \mathfrak{es}_{\sigma\psi} & \mathfrak{es}_{\sigma\beta} & \mathfrak{es}_{\sigma\sigma} & \mathfrak{es}_{\sigma\varepsilon} & \mathfrak{es}_{\sigma\gamma} & \mathfrak{es}_{\sigma\pi} \\ 7\,(\varepsilon) & \mathfrak{es}_{\varepsilon 0} & \mathfrak{es}_{\varepsilon i} & \mathfrak{es}_{\varepsilon\psi} & \mathfrak{es}_{\varepsilon\beta} & \mathfrak{es}_{\varepsilon\sigma} & \mathfrak{es}_{\varepsilon\varepsilon} & \mathfrak{es}_{\varepsilon\gamma} & \mathfrak{es}_{\varepsilon\pi} \\ 8\,(\gamma) & \mathfrak{es}_{\gamma 0} & \mathfrak{es}_{\gamma i} & \mathfrak{es}_{\gamma\psi} & \mathfrak{es}_{\gamma\beta} & \mathfrak{es}_{\gamma\sigma} & \mathfrak{es}_{\gamma\varepsilon} & \mathfrak{es}_{\gamma\gamma} & \mathfrak{es}_{\gamma\pi} \\ 9\,(\pi) & \mathfrak{es}_{\pi 0} & \mathfrak{es}_{\pi i} & \mathfrak{es}_{\pi\psi} & \mathfrak{es}_{\pi\beta} & \mathfrak{es}_{\pi\sigma} & \mathfrak{es}_{\pi\varepsilon} & \mathfrak{es}_{\pi\gamma} & \mathfrak{es}_{\pi\pi} \end{array}\right]$$

Indices: $M, N = 0, 1, \ldots, 9$. Spatiotemporal indices: $\mu, \nu = 0, 1, 2, 3$. Ethosystemic indices: $m, n = 4, 5, 6, 7, 8, 9$.

Dimension key: $0 = t$ (temporal), $1{-}3 = x^i$ (spatial), $4 = \psi$ (psychological), $5 = \beta$ (biological), $6 = \sigma$ (social-cultural), $7 = \varepsilon$ (economic), $8 = \gamma$ (geographic), $9 = \pi$ (political).

Block Structure

$$\mathfrak{es}_{MN} = \left[\begin{array}{cc} \overbrace{\mathfrak{es}_{\mu\nu}}^{\text{spatiotemporal geometry}} & \overbrace{\mathfrak{es}_{\mu n}}^{\text{cross-coupling}} \\ \underbrace{\mathfrak{es}_{m\nu}}_{\text{cross-coupling}} & \underbrace{\mathfrak{es}_{mn}}_{\text{ethosystemic geometry}} \end{array}\right] \quad \begin{array}{l} \mu,\nu{=}0,1,2,3 \\ m,n{=}4,5,6,7,8,9 \end{array}$$

Crucially, $\mathfrak{es}_{MN}$ is a *field*: its components vary across the manifold. The ethosystemic geometry at one point in the ethosystemic medium may differ from the geometry at another. A community's

[58] The $4+6$ structure of the ethosystemic manifold is modular: if future work reveals that some aspect of the ethosystemic medium has dynamics genuinely irreducible to the six identified dimensions and their couplings, the manifold can be expanded accordingly without altering the tensor-field architecture. Conversely, if some dimensions prove to be reducible to others, the manifold can be simplified. The ten-dimensional model is the best current hypothesis about the ethosystemic dimensionality, motivated by the independent analysis of what the irreducible dimensions of ethical environmental variation are.

psychological, biological, social-cultural, economic, geographic, and political configuration (and the couplings among these dimensions) shift over time and vary across space and ethosystemic dimensions (e.g., level of wealth and different intersectional identity). Moreover, the ethosystemic metric is dynamical: it co-evolves with the ethical fields ($\mathcal{G}$, $\mathcal{R}$, $\mathcal{V}$) that propagate through it, just as the spacetime metric in general relativity co-evolves with the matter fields that propagate through spacetime.

This ethosystemic tensor decomposes uniquely into symmetric and antisymmetric parts:

$$\mathfrak{es}_{MN} \;=\; \underbrace{\bar{\mathfrak{es}}_{(MN)}}_{\text{ethosystemic geometry}} \;+\; \underbrace{\mathfrak{a}_{[MN]}}_{\text{ethosystemic anisotropy}} \tag{7}$$

where

$$\bar{\mathfrak{es}}_{(MN)} = \tfrac{1}{2}(\mathfrak{es}_{MN} + \mathfrak{es}_{NM})\,, \qquad \bar{\mathfrak{es}}_{(MN)} = \bar{\mathfrak{es}}_{(NM)}, \tag{8}$$

$$\mathfrak{a}_{[MN]} = \tfrac{1}{2}(\mathfrak{es}_{MN} - \mathfrak{es}_{NM})\,, \qquad \mathfrak{a}_{[MN]} = -\mathfrak{a}_{[NM]}. \tag{9}$$

The generalized ethosystemic interval between infinitesimally separated points on the manifold is:

$$d\mathfrak{s}^2 \;=\; \underbrace{\bar{\mathfrak{es}}_{(MN)}\, d\xi^M\, d\xi^N}_{\text{symmetric (quadratic) distance}} \;+\; \underbrace{\mathfrak{a}_{[MN]}\, d\xi^M \wedge d\xi^N}_{\text{anisotropic (directional) correction}} \tag{10}$$

The first term is the standard quadratic form familiar from Riemannian geometry: it measures the direction-independent "distance" between ethosystemic configurations and is determined entirely by the symmetric component $\bar{\mathfrak{es}}_{(MN)}$. The second term is the anisotropic correction: it involves the wedge product $d\xi^M \wedge d\xi^N = d\xi^M \otimes d\xi^N - d\xi^N \otimes d\xi^M$ (the natural antisymmetric product of coordinate differentials) and is determined by the anisotropy tensor $\mathfrak{a}_{[MN]}$. This term is *direction-dependent*: it changes sign when the direction of traversal is reversed ($d\xi^M \wedge d\xi^N = -d\xi^N \wedge d\xi^M$), encoding the fact that moving through the ethosystemic medium in one direction is not equivalent to moving through it in the reverse direction.

The anisotropic correction is invisible to purely pointwise measures of ethosystemic distance but becomes visible—and ethically consequential—when one considers *directed transitions* through the medium: processes that move from one ethosystemic configuration to another along a specific path with a specific orientation. This is precisely the kind of probe that ethical evaluation involves: the ethical fields $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ do not merely register static configurations but track directed transitions through the ethical landscape, and these transitions couple to the full ethosystemic tensor—both its symmetric geometry and its antisymmetric anisotropy.

The symmetric component $\bar{\mathfrak{es}}_{(MN)}$—the *ethosystemic geometry* proper—encodes the mutual, non-directional coupling between dimensions: the structure that would remain if every coupling were perfectly reciprocal. It determines "distances," angles, and the curvature of the ethosystemic manifold.

The antisymmetric component $\mathfrak{a}_{[MN]}$—the *ethosystemic anisotropy tensor*—encodes the directional asymmetry of the couplings: the extent to which the influence of dimension M on dimension N differs from the influence of N on M. Where $\mathfrak{a}_{[MN]} = 0$, the coupling is reciprocal and the medium is locally isotropic in those dimensions. Where $\mathfrak{a}_{[MN]} \neq 0$, the medium exhibits structural anisotropy, i.e., a preferred directionality of influence that reflects asymmetries of power, vulnerability, and causal structure in the ethosystemic medium.[59]

[59]The decomposition of a general background tensor into symmetric and antisymmetric parts has a precise

The ethical fields $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ propagate through and couple to the *full* ethosystemic tensor $\mathfrak{es}_{MN}$, not merely its symmetric part. They respond to both the geometry and the anisotropy of the medium: the evaluative content of an ethical situation depends not only on how strongly the ethosystemic dimensions are coupled (encoded in $\bar{\mathfrak{es}}_{(MN)}$) but on the *directionality* of those couplings (encoded in $\mathfrak{a}_{[MN]}$). A community in which economic structures shape political possibilities far more strongly than political decisions reshape economic structures ($\mathfrak{a}_{[\varepsilon\pi]} \gg 0$) presents a different ethical landscape from one in which the coupling is reciprocal ($\mathfrak{a}_{[\varepsilon\pi]} \approx 0$), even if the overall strength of the econo-political coupling ($\bar{\mathfrak{es}}_{(\varepsilon\pi)}$) is the same in both cases. This gives formal expression to a central insight of critical social theory and intersectional analysis: that structural injustice is not merely a matter of bad outcomes (which would be captured by the diagonal entries of the ethical field tensors) or even of antagonistic coupling between ethical dimensions (which would be captured by the off-diagonal entries of $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$) but of *asymmetric structure in the medium itself*—anisotropies in the ethosystemic metric that produce systematically different conditions for different agents depending on their position in the medium (e.g., their level of income, their particular psychological and bodily condition, their intersectional identity). Crucially, these anisotropies lead to systematic and systemic asymmetries in *how ethical fields propagate* through the ethosystem. The same good, the same right, the same virtue does not propagate with equal reach, equal efficacy, or equal uptake across an anisotropic ethosystemic medium. A right that is formally recognized may propagate into effective social reality for some intersectional identities but not for others; a good that arises in an economically privileged region of the ethosystem may propagate readily into political influence, while the same good arising in a marginalized region is absorbed, deflected, or dissipated by the medium's anisotropy before it can reach the political dimension. The anisotropy tensor $\mathfrak{a}_{[MN]}$ is the formal object that encodes this structural-level asymmetry in both its aspects: the static asymmetry in the medium's geometry *and* the dynamic asymmetry in how ethical fields propagate through it. Structural injustice, on this analysis, is anisotropy in the ethosystemic metric, anisotropy that does not merely describe asymmetric background conditions but that *actively distorts* the propagation of goodness, rightness, and virtue through the ethosystemic medium, systematically advantaging some positions in the ethosystem and disadvantaging others.[60]

If the argument of this paper succeeds, its contribution is twofold. At the level of substantive moral theory, it provides a non-reductive unification of the three fundamental ethical dimensions, a structural characterization of ethical dilemmas and moral synergies, and an account of the emergent dynamics (virtuous and vicious spirals) that arise from the coupling among dimensions, results not available from within any single ethical tradition and not articulable without the tensor-field formalism. At the level of methodology, it demonstrates that the structural analogy between physics and ethics, when pursued with formal care and honest attention to disanalogies, is not a loose metaphor but a source of determinate philosophical results, results that could not have been obtained without the tensor-field vocabulary and that open a structured research program extending from the characterization of ethical coupling operators to the dynamics of embodied and

analog in string theory, where the background fields in which a string propagates include both the spacetime metric g_{MN} (symmetric) and the Kalb-Ramond 2-form field B_{MN} (antisymmetric). The string's worldsheet action couples to the combination $g_{MN} + B_{MN}$—a general rank-2 tensor—and the Kalb-Ramond field encodes directional structure (torsion, flux) that the metric alone cannot capture. The ethosystemic anisotropy tensor $\mathfrak{a}_{[MN]}$ plays a structurally analogous role: it encodes the directional structure of the ethosystemic medium that the symmetric geometry $\bar{\mathfrak{es}}_{(MN)}$ alone cannot represent. In both cases, the full dynamics requires the general tensor, not merely its symmetric part.

[60] I develop the ethosystem model further in other work (Sanchez Borboa, manuscript C).

embedded ethical systems to the integration of form and matter within a unified ethical field theory.

The three fundamental ethical dimensions—the good, the right, and virtue

—have been recognized since antiquity. That they are interdependent has been understood since at least Aristotle. What has been lacking is a formal framework that represents their interdependence *as* interdependence: not as reduction of some to others, not as mere coexistence without structure, but as dynamic coupling among co-fundamental fields whose interactions generate the richness, the tragedy, and the promise of the moral life. The tensor-field model provides that framework. I hope to have shown that its development has just begun.[61]

[61]I thank Melissa Barry, Bryan Chambliss, and Robert Wallace for helpful discussions concerning how the ethical field model can illuminate everyday ethical phenomenology by, e.g., understanding the good will people bring to interactions in many contexts as positive ethical radiation (which vividly contrasts with a lack of good will or the negative ethical radiation of ill will). These discussions inspired me to keep developing this project. I also thank the students in my Introduction to Ethics Course at Loyola University of Chicago, whose engagement provided evidence of the conceptual tractability and pedagogical fruitfulness of the ethical field model. I thank Chris Howard for helpful conversations concerning the need to recognize multiple ethical dimensions to fully account for the normative terrain. I thank Fred D'Agostino, and Jerry Gaus's and Jenann Ismael's Complexity seminar for helpful feedback concerning the project of providing a common formal framework in which to articulate different normative ethical positions early on in the project. I thank Ernesto Ulloa for early feedback concerning the Rawls-Santian equations and reassurance that the basic idea for the formalization was sound. The idea to formally describing ethical fields and their coupled dynamics by thinking of them as analogous to the formally described physical ones (and exploring disanalogies between physics and ethics as opportunities for innovation) was inspired by Stephon Alexander's *The Jazz of Physics* (2016), which compellingly argues for the fruitfulness of drawing interdisciplinary analogies between music and physics. Jordan Ellenberg's point that new geometry serves as an independent locus of rational authority in *Shape* helped inspire the idea of crafting a (formally specified) ethical formal ideal (i.e., syntegrity) that aspires to an objective, structural authority because it is inherently geometric (specifying the condition under which all ethical dimensions jointly flourish due to their positive couplings) (2021). G.E. Moore's discussion of organic unities (wholes whose value is not equal to that of the sum of their parts) in *Principia Ethica* (1903) inspired me to take non-additive ethical relations seriously. In this paper's formalization, the synergistic or antagonistic off-diagonal couplings of ethical tensor fields encode these non-additive relations.

Appendix: Proposed Structural Field Equations

This appendix makes explicit the dynamical content of the coupling principles developed in §3.6 by formulating them as proposed field equations governing the co-evolution of the axiological, deontic, and aretaic tensor fields, the Rawls-Santian equations.[62] The equations are structural: they specify the formal architecture of the inter-field coupling (which fields drive which, through what mathematical operations, subject to what modulations by the ethosystemic medium) without claiming the quantitative precision of their electromagnetic analogs. Their precise form (in particular, the determination of the coupling operators $\mathcal{C}^{A\to B}$, the functional dependence of the permeabilities $\mathfrak{g}(\xi), \mathfrak{r}(\xi), \mathfrak{v}(\xi)$ on the ethosystemic metric, and the relationship between the damping and driving coefficients and the configuration of the ethosystemic medium) is an open question for future work, one that lies at the frontier of the research program. What the equations provide in their present form is threefold: a formal specification of the multiplicative coupling structure argued for in §3.3, making the non-additive character of the inter-field dynamics (§4.6) mathematically explicit; a demonstration that the tensor-field framework generates specific dynamical predictions, including an ethical wave equation whose solutions describe self-propagating coupled oscillations of the $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ fields (ethical radiation) with a characteristic propagation speed determined by the triple permeability product of the ethosystemic medium; and a natural connection to the gauge-theoretic apparatus of the companion papers, achieved by promoting the ordinary derivatives in the equations to gauge-covariant derivatives incorporating the ethical connection field $\mathcal{C}_M$. The notation is consistent with the main text and the companion papers throughout.

A.1 Preliminary Notation and Conventions

I adopt the notation of the companion papers throughout. The fundamental objects are:

- W: the domain of ethically relevant cases (the ethical case space).
- $\xi^M = (x^0, x^1, x^2, x^3, w^4, w^5, w^6, w^7, w^8, w^9)$: coordinates on the 10-dimensional ethosystemic manifold, where $M = 0, 1, \ldots, 9$. The first four are spatiotemporal; the remaining six are properly ethosystemic (psychological, biological, social-cultural, economic, geographic, political).
- $\mathcal{G}^{\alpha\beta}(\xi)$: the axiological field, $\alpha, \beta = 1, \ldots, n_{\mathcal{G}}$.
- $\mathcal{R}^{\rho\omega}(\xi)$: the deontic field, $\rho, \omega = 1, \ldots, n_{\mathcal{R}}$.
- $\mathcal{V}^{\nu\kappa}(\xi)$: the aretaic field, $\nu, \kappa = 1, \ldots, n_{\mathcal{V}}$.
- $\mathfrak{es}_{MN}(\xi)$: the ethosystemic metric tensor field.
- $\mathcal{C}_M(\xi)$: the ethical connection field (the good will), taking values in the Lie algebra of the ethical gauge group.

[62] The designation acknowledges the two intellectual debts that structure the equations: to Rawls, whose characterization of moral theory as the study of how the good, the right, and moral worth may be arranged to form different moral structures provides the tripartite field structure that the equations govern; and to the present author, who develops the tensor-field formalism, the dynamic coupling architecture, and the ethosystemic embedding within which the equations operate.

Each ethical field $\mathcal{F} \in \{\mathcal{G}, \mathcal{R}, \mathcal{V}\}$ carries two kinds of indices: *internal ethical indices* (Greek: $\alpha\beta$, $\rho\omega$, $\nu\kappa$) encoding the intra-dimensional tensor structure developed in the main paper, and *ethosystemic indices* ($M, N, \ldots$) encoding how the field propagates through the ethosystemic manifold. The internal ethical indices are retained throughout, as they represent the full rank-2 tensor structure of each field—the diagonal entries encoding dimensional content and the off-diagonal entries encoding the synergies and antagonisms between sub-dimensions that are the central formal innovation of the main paper. The Rawls-Santian equations govern both levels of structure simultaneously.

Here, I use the following additional notation:

- ∇_M: the covariant derivative on the ethosystemic manifold (compatible with $\mathfrak{es}_{MN}$).
- $\mathfrak{r}(\xi)$, $\mathfrak{g}(\xi)$, $\mathfrak{v}(\xi)$: the deontic, axiological, and aretaic *permeabilities* of the ethosystemic medium—field-valued functions encoding how readily each ethical field propagates through the ethosystem at each point. These are the ethical analogs of the electromagnetic permittivity ε_0 and permeability μ_0.
- $\Omega_{\mathcal{G}}^{\alpha\beta}(\xi)$, $\Omega_{\mathcal{R}}^{\rho\omega}(\xi)$, $\Omega_{\mathcal{V}}^{\nu\kappa}(\xi)$: the axiological, deontic, and aretaic *charge density tensors*—the sources (and sinks) of the respective ethical fields.
- $\mathcal{J}_{\mathcal{G}}^{\alpha\beta,M}(\xi)$, $\mathcal{J}_{\mathcal{R}}^{\rho\omega,M}(\xi)$, $\mathcal{J}_{\mathcal{V}}^{\nu\kappa,M}(\xi)$: the axiological, deontic, and aretaic *current tensors*—the ethosystemic flow of ethical charges.
- $\mathcal{C}^{\mathcal{A}\to\mathcal{B}}$: the coupling operator from field $\mathcal{A}$ to field $\mathcal{B}$ (a tensor-valued map from $\mathbb{R}^{n_{\mathcal{A}}\times n_{\mathcal{A}}}$ to $\mathbb{R}^{n_{\mathcal{B}}\times n_{\mathcal{B}}}$, itself field-valued over ξ).
- $\chi^{\mathcal{A}\to\mathcal{B}}(\xi) \geq 0$: the *damping coefficient* for the influence of field $\mathcal{A}$ on field $\mathcal{B}$, encoding the dissipation of ethical energy due to finite material conditions in the ethosystemic medium.
- $\sigma^{\mathcal{A}\to\mathcal{B}}(\xi) \geq 0$: the *driving coefficient* for the influence of field $\mathcal{A}$ on field $\mathcal{B}$, encoding the injection of ethical energy by agential intervention.

Remark 1 (Effective permeability)**.** *In each coupling equation, the base permeability, the damping coefficient, and the driving coefficient combine into an* effective permeability*:*

$$\tilde{\mathfrak{f}}^{\mathcal{A}\to\mathcal{B}}(\xi) \;=\; \mathfrak{f}_{\mathcal{A}}(\xi) - \chi^{\mathcal{A}\to\mathcal{B}}(\xi) + \sigma^{\mathcal{A}\to\mathcal{B}}(\xi) \tag{11}$$

where $\mathfrak{f}_{\mathcal{A}} \in \{\mathfrak{g}, \mathfrak{r}, \mathfrak{v}\}$ is the base permeability of field $\mathcal{A}$. When $\chi^{\mathcal{A}\to\mathcal{B}} > \sigma^{\mathcal{A}\to\mathcal{B}}$, the medium is dissipative for that coupling direction (ethical energy is lost faster than agents replenish it); when $\sigma^{\mathcal{A}\to\mathcal{B}} > \chi^{\mathcal{A}\to\mathcal{B}}$, the medium is generative (agents are actively amplifying the coupling); when $\chi^{\mathcal{A}\to\mathcal{B}} = \sigma^{\mathcal{A}\to\mathcal{B}}$, the medium is in dynamic equilibrium for that coupling.

A.2 The Source Equations (Ethical Gauss's Laws)

The source equations specify how ethical charge densities generate the ethical fields, modified by damping and driving. They are the ethical analogs of Gauss's law for electricity ($\nabla \cdot \mathbf{E} = \rho/\varepsilon_0$) and the (symmetrically modified) Gauss's law for magnetism.

Definition 6 (Source Equations)**.** *At each point ξ in the ethosystemic manifold:*

$$\nabla_M \mathcal{G}^{\alpha\beta,M}(\xi) = \mathfrak{g}(\xi)\,\Omega_{\mathcal{G}}^{\alpha\beta}(\xi) \;-\; \chi_{\mathrm{src}}^{\mathcal{G}}(\xi)\,\nabla_M \mathcal{G}^{\alpha\beta,M}(\xi) \;+\; \sigma_{\mathrm{src}}^{\mathcal{G}}(\xi)\,\mathcal{G}^{\alpha\beta}(\xi) \tag{12}$$

$$\nabla_M \mathcal{R}^{\rho\omega,M}(\xi) = \mathfrak{r}(\xi)\,\Omega_{\mathcal{R}}^{\rho\omega}(\xi) \;-\; \chi_{\mathrm{src}}^{\mathcal{R}}(\xi)\,\nabla_M \mathcal{R}^{\rho\omega,M}(\xi) \;+\; \sigma_{\mathrm{src}}^{\mathcal{R}}(\xi)\,\mathcal{R}^{\rho\omega}(\xi) \tag{13}$$

$$\nabla_M \mathcal{V}^{\nu\kappa,M}(\xi) = \mathfrak{v}(\xi)\,\Omega_{\mathcal{V}}^{\nu\kappa}(\xi) \;-\; \chi_{\mathrm{src}}^{\mathcal{V}}(\xi)\,\nabla_M \mathcal{V}^{\nu\kappa,M}(\xi) \;+\; \sigma_{\mathrm{src}}^{\mathcal{V}}(\xi)\,\mathcal{V}^{\nu\kappa}(\xi) \tag{14}$$

where the subscript 'src' distinguishes the damping and driving coefficients appearing in the source equations (which modulate field generation) from those appearing in the coupling equations (which modulate inter-field transmission, and which carry directional superscripts $\mathcal{A} \to \mathcal{B}$).

Remark 2 (Interpretation)**.** *Each source equation states that the ethosystemic divergence of an ethical field (the net "flux" of that field out of an infinitesimal ethosystemic region) is determined by three contributions:*

(i) The charge density term *($\mathfrak{f}\,\Omega_{\mathcal{F}}$): ethical sources and sinks generate the field, modulated by the medium's permeability.*

(ii) The damping term *($-\chi_{\mathrm{src}}^{\mathcal{F}}\,\nabla_M \mathcal{F}^M$): the ethosystemic medium dissipates ethical energy proportionally to the field's current divergence, reducing the field's effective reach.*

(iii) The driving term *($+\sigma_{\mathrm{src}}^{\mathcal{F}}\,\mathcal{F}$): agential intervention sustains or amplifies the field proportionally to the field's current configuration.*[63]

The source equations can be rewritten in effective form by collecting the divergence terms:

$$\big(1 + \chi_{\mathrm{src}}^{\mathcal{F}}(\xi)\big)\,\nabla_M \mathcal{F}^M(\xi) \;=\; \mathfrak{f}(\xi)\,\Omega_{\mathcal{F}}(\xi) \;+\; \sigma_{\mathrm{src}}^{\mathcal{F}}(\xi)\,\mathcal{F}(\xi) \tag{15}$$

which makes transparent that damping effectively reduces the medium's permeability to source generation (it takes more charge density to produce the same field divergence), while driving introduces a self-sustaining feedback (the field's own configuration contributes to its maintenance).

A.3 The Dynamic Coupling Equations (Ethical Faraday-Ampère Laws)

The coupling equations specify how changes in each ethical field generate changes in the other two. They are the ethical analogs of Faraday's law ($\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$) and Ampère's law with Maxwell's displacement current ($\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0\, \partial \mathbf{E}/\partial t$).

The critical structural difference from the electromagnetic case is that each ethical field couples to *two* other fields rather than one, and the coupling is *multiplicative* rather than additive, reflecting the non-additive character of the ethical dynamics argued for in the main paper (§4.6).

[63] The asymmetric form of the damping and driving terms reflects the dominant mechanisms: medium resistance operates primarily on the field's propagation (proportional to divergence), while agential reinforcement operates primarily on the field's presence (proportional to configuration). A fully general treatment would include all four combinations, i.e., medium damping and agential driving of both propagation and presence, yielding effective coefficients that modulate the divergence and the field value independently. The present form is the minimal extension that captures the distinctive features of the ethical domain; the four-term generalization is a natural refinement for future work.

Definition 7 (Dynamic Coupling Equations)**.** *At each point ξ in the ethosystemic manifold:*

$$\partial_t \mathcal{R}^{\rho\omega}(\xi) \;=\; \mathfrak{r}(\xi)\,\Big\{\,\mathcal{C}^{\mathcal{G}\to\mathcal{R}}\Big[\,\mathcal{J}_{\mathcal{G}}^{\alpha\beta,M}(\xi) \;+\; \tilde{\mathfrak{g}}^{\,\mathcal{G}\to\mathcal{R}}(\xi)\,\partial_t\,\mathcal{G}^{\alpha\beta}(\xi)\,\Big]\;\cdot\;\mathcal{C}^{\mathcal{V}\to\mathcal{R}}\Big[\,\mathcal{J}_{\mathcal{V}}^{\nu\kappa,M}(\xi) \;+\; \tilde{\mathfrak{v}}^{\,\mathcal{V}\to\mathcal{R}}(\xi)\,\partial_t\,\mathcal{V}^{\nu\kappa}(\xi)\,\Big]\,\Big\} \tag{16}$$

$$\partial_t \mathcal{G}^{\alpha\beta}(\xi) \;=\; \mathfrak{g}(\xi)\,\Big\{\,\mathcal{C}^{\mathcal{V}\to\mathcal{G}}\Big[\,\mathcal{J}_{\mathcal{V}}^{\nu\kappa,M}(\xi) \;+\; \tilde{\mathfrak{v}}^{\,\mathcal{V}\to\mathcal{G}}(\xi)\,\partial_t\,\mathcal{V}^{\nu\kappa}(\xi)\,\Big]\;\cdot\;\mathcal{C}^{\mathcal{R}\to\mathcal{G}}\Big[\,\mathcal{J}_{\mathcal{R}}^{\rho\omega,M}(\xi) \;+\; \tilde{\mathfrak{r}}^{\,\mathcal{R}\to\mathcal{G}}(\xi)\,\partial_t\,\mathcal{R}^{\rho\omega}(\xi)\,\Big]\,\Big\} \tag{17}$$

$$\partial_t \mathcal{V}^{\nu\kappa}(\xi) \;=\; \mathfrak{v}(\xi)\,\Big\{\,\mathcal{C}^{\mathcal{R}\to\mathcal{V}}\Big[\,\mathcal{J}_{\mathcal{R}}^{\rho\omega,M}(\xi) \;+\; \tilde{\mathfrak{r}}^{\,\mathcal{R}\to\mathcal{V}}(\xi)\,\partial_t\,\mathcal{R}^{\rho\omega}(\xi)\,\Big]\;\cdot\;\mathcal{C}^{\mathcal{G}\to\mathcal{V}}\Big[\,\mathcal{J}_{\mathcal{G}}^{\alpha\beta,M}(\xi) \;+\; \tilde{\mathfrak{g}}^{\,\mathcal{G}\to\mathcal{V}}(\xi)\,\partial_t\,\mathcal{G}^{\alpha\beta}(\xi)\,\Big]\,\Big\} \tag{18}$$

where the effective permeabilities are:

$$\tilde{\mathfrak{g}}^{\,\mathcal{G}\to\mathcal{B}}(\xi) = \mathfrak{g}(\xi) - \chi^{\mathcal{G}\to\mathcal{B}}(\xi) + \sigma^{\mathcal{G}\to\mathcal{B}}(\xi) \tag{19}$$

$$\tilde{\mathfrak{r}}^{\,\mathcal{R}\to\mathcal{B}}(\xi) = \mathfrak{r}(\xi) - \chi^{\mathcal{R}\to\mathcal{B}}(\xi) + \sigma^{\mathcal{R}\to\mathcal{B}}(\xi) \tag{20}$$

$$\tilde{\mathfrak{v}}^{\,\mathcal{V}\to\mathcal{B}}(\xi) = \mathfrak{v}(\xi) - \chi^{\mathcal{V}\to\mathcal{B}}(\xi) + \sigma^{\mathcal{V}\to\mathcal{B}}(\xi) \tag{21}$$

for any target field $\mathcal{B} \in \{\mathcal{G}, \mathcal{R}, \mathcal{V}\}$, and $\cdot$ denotes the tensorial product whose precise form is determined by the coupling operators.

Remark 3 (Interpretation)**.** *Each coupling equation states that the temporal evolution of an ethical field at a point ξ is determined by the* multiplicative interaction *of the coupled contributions from the other two fields. Consider equation* (16)*: the rate of change of the deontic field $\mathcal{R}$ is driven by the product of:*

(i) The axiological contribution: the current flow of axiological charges ($\mathcal{J}_{\mathcal{G}}$) plus the temporal change in the axiological field ($\partial_t \mathcal{G}$), the latter modulated by the effective axiological permeability $\tilde{\mathfrak{g}}^{\,\mathcal{G}\to\mathcal{R}}$, all transformed by the coupling operator $\mathcal{C}^{\mathcal{G}\to\mathcal{R}}$ into deontic-field-valued content.

(ii) The aretaic contribution: the current flow of aretaic charges ($\mathcal{J}_{\mathcal{V}}$) plus the temporal change in the aretaic field ($\partial_t \mathcal{V}$), similarly modulated and transformed.

The multiplicative structure encodes the non-additive character of the ethical dynamics: when both contributions are positive, their product amplifies faster than their sum (virtuous spiral); when one contribution is severely degraded, the product propagates the damage more aggressively than an additive model would predict (vicious spiral onset). This is consonant with the nonabelian character of the ethical connection field argued for in the Interacting Ethical Fields paper (§4.8.3), where products of field components arise from the noncommutativity of the Lie algebra generators.

Remark 4 (The role of damping and driving in the coupling equations)**.** *Within each coupling equation, the effective permeability $\tilde{\mathfrak{f}}^{\,\mathcal{A}\to\mathcal{B}}$ modulates the contribution of field $\mathcal{A}$'s temporal change to the evolution of field $\mathcal{B}$. Three regimes are significant:*

(i) Dissipative regime *($\chi^{\mathcal{A}\to\mathcal{B}} > \sigma^{\mathcal{A}\to\mathcal{B}}$): $\tilde{\mathfrak{f}}^{\,\mathcal{A}\to\mathcal{B}} < \mathfrak{f}_{\mathcal{A}}$. The medium attenuates the coupling—changes in $\mathcal{A}$ generate weaker changes in $\mathcal{B}$ than the base permeability would predict. This models situations where material conditions (economic deprivation, institutional decay, environmental degradation) erode the inter-field coupling.*

(ii) Generative regime *($\sigma^{\mathcal{A}\to\mathcal{B}} > \chi^{\mathcal{A}\to\mathcal{B}}$): $\tilde{\mathfrak{f}}^{\mathcal{A}\to\mathcal{B}} > \mathfrak{f}_{\mathcal{A}}$. Agential intervention amplifies the coupling—changes in $\mathcal{A}$ generate stronger changes in $\mathcal{B}$ than the base permeability alone would produce. This models situations where active moral practice (education, institutional design, community building) strengthens the inter-field coupling.*

(iii) Dynamic equilibrium *($\sigma^{\mathcal{A}\to\mathcal{B}} = \chi^{\mathcal{A}\to\mathcal{B}}$): $\tilde{\mathfrak{f}}^{\mathcal{A}\to\mathcal{B}} = \mathfrak{f}_{\mathcal{A}}$. The driving exactly compensates the damping, and the coupling operates at its base strength. This is the condition toward which well-maintained moral communities tend.*

These regimes have no electromagnetic analog, since electromagnetic fields propagate through the vacuum without dissipation and without the possibility of agential amplification. Their presence in the ethical equations reflects the agent-dependent character of the ethical field discussed in the Interacting Ethical Fields paper (§5.3): the ethical fields are at least partly constituted by agents whose activity can sustain, amplify, or neglect them.

A.4 The Ethical Wave Equation

Just as Maxwell's coupling equations yield the electromagnetic wave equation (predicting the existence and speed of light), the Rawls-Santian coupling equations yield an ethical wave equation predicting the existence and propagation speed of *ethical radiation*—self-sustaining coupled oscillations of the $\mathcal{G}$, $\mathcal{R}$, and $\mathcal{V}$ fields.

Definition 8 (Ethical Wave Equation)**.** *Let $\phi^{\pi\eta}(\xi)$ denote the holistic ethical field (a component of the block tensor $\mathscr{E}^{\mathcal{H}}$). In the linearized, source-free, equilibrium regime (where the damping and driving balance and the coupling operators are approximately constant), the coupling equations yield:*

$$\nabla^2_{\mathrm{ES}}\,\phi^{\pi\eta}(\xi) \;-\; \mathfrak{r}(\xi)\,\mathfrak{g}(\xi)\,\mathfrak{v}(\xi)\;\partial_t^2\,\phi^{\pi\eta}(\xi) \;=\; \mathcal{S}^{\pi\eta}(\xi) \tag{22}$$

where:

- $\nabla^2_{\mathrm{ES}} = \mathfrak{es}^{MN}\nabla_M\nabla_N$ *is the ethosystemic Laplacian (the covariant generalization of the Laplacian to the curved ethosystemic manifold),*
- $\mathfrak{r}\,\mathfrak{g}\,\mathfrak{v}$ *is the* triple permeability product *(the ethical analog of $\mu_0\varepsilon_0$), and*
- $\mathcal{S}^{\pi\eta}(\xi)$ *is a source term encoding the contribution of ethical charges and currents.*

Remark 5 (The speed of ethical light)**.** *The wave equation* (22) *implies a characteristic propagation speed:*

$$c_{\mathrm{eth}}(\xi) \;=\; \frac{1}{\sqrt{\mathfrak{r}(\xi)\,\mathfrak{g}(\xi)\,\mathfrak{v}(\xi)}} \tag{23}$$

This is the speed of ethical light*: the maximum speed at which coupled ethical field disturbances propagate through the ethosystemic medium. Unlike the physical speed of light $c = 1/\sqrt{\mu_0\varepsilon_0}$ (a universal constant), the speed of ethical light is* field-valued*—it varies across the ethosystemic manifold, depending on the local permeabilities. In regions of high ethical permeability (where the medium is highly conducive to ethical field propagation), c_{eth} is lower (disturbances propagate more slowly but with greater intensity, as the medium absorbs and re-radiates the ethical content more thoroughly); in regions of low permeability (where the medium resists ethical field propagation), c_{eth} is higher (disturbances propagate more rapidly but with less local engagement).*

The triple product $\mathfrak{r}\,\mathfrak{g}\,\mathfrak{v}$ reflects the three-field structure of the ethical domain: ethical radiation requires the coupled oscillation of all three fields, and its propagation speed depends on the medium's receptivity to each. This is a structural difference from the electromagnetic case, where the two-field product $\mu_0\varepsilon_0$ suffices.

A.5 The Holistic Block Form

The source equations and coupling equations can be assembled into a single system governing the holistic block tensor $\mathscr{E}^{\mathcal{H}}$ introduced in the main paper (Definition 5). Partitioning $\mathscr{E}^{\mathcal{H}}$ into its diagonal blocks ($\mathcal{G}$, $\mathcal{R}$, $\mathcal{V}$) and off-diagonal blocks ($\mathcal{GR}$, $\mathcal{GV}$, $\mathcal{RG}$, $\mathcal{RV}$, $\mathcal{VG}$, $\mathcal{VR}$), the Rawls-Santian equations take the compact form:

Definition 9 (Block Form of the Rawls-Santian Equations)**.**

$$\boxed{\partial_t \mathscr{E}^{\mathcal{H}}_{\mathcal{IJ}}(\xi) \;=\; \mathfrak{F}_{\mathcal{I}}(\xi)\,\Big\{\,\mathcal{C}^{\mathcal{K}\to\mathcal{I}}\Big[\,\mathcal{J}_{\mathcal{K}}(\xi) \;+\; \tilde{\mathfrak{F}}^{\,\mathcal{K}\to\mathcal{I}}(\xi)\,\partial_t\,\mathscr{E}^{\mathcal{H}}_{\mathcal{KK}}(\xi)\,\Big]\;\cdot\;\mathcal{C}^{\mathcal{L}\to\mathcal{I}}\Big[\,\mathcal{J}_{\mathcal{L}}(\xi) \;+\; \tilde{\mathfrak{F}}^{\,\mathcal{L}\to\mathcal{I}}(\xi)\,\partial_t\,\mathscr{E}^{\mathcal{H}}_{\mathcal{LL}}(\xi)\,\Big]\,\Big\}} \tag{24}$$

where $\mathcal{I},\mathcal{J}\in\{\mathcal{G},\mathcal{R},\mathcal{V}\}$, and for each $\mathcal{I}$, the fields $\mathcal{K}$ and $\mathcal{L}$ are the other two members of $\{\mathcal{G},\mathcal{R},\mathcal{V}\}\setminus\{\mathcal{I}\}$, and $\mathfrak{F}_{\mathcal{I}}$ is the permeability of field $\mathcal{I}$.

Remark 6. *Equation* (24) *makes explicit that the Rawls-Santian system is a* single coupled dynamical system *on the holistic block tensor $\mathscr{E}^{\mathcal{H}}$, in which each diagonal block's evolution is driven by the multiplicative interaction of the other two diagonal blocks (transformed by the coupling operators and modulated by the effective permeabilities). The off-diagonal blocks of $\mathscr{E}^{\mathcal{H}}$—the inter-dimensional coupling terms—are determined by the coupling operators $\mathcal{C}^{\mathcal{A}\to\mathcal{B}}$ acting on the diagonal blocks. The entire system co-evolves as a unit, which is the formal content of the claim that the three ethical dimensions are dynamically coupled aspects of a single ethical field structure.*

A.6 Connection to the Gauge-Theoretic Apparatus

The Rawls-Santian equations as formulated in §§1–2 use ordinary (partial or covariant) derivatives. When the connection field $\mathcal{C}_M$ (the good will, as identified in the Geometry paper) is introduced, the ordinary derivatives are promoted to *gauge-covariant derivatives*:

$$\nabla_M \;\longrightarrow\; D_M \;=\; \nabla_M + \mathcal{C}_M \tag{25}$$

and the Rawls-Santian equations become:

Definition 10 (Gauge-Covariant Rawls-Santian Equations)**.**

$$D_M\,\mathcal{G}^{\alpha\beta,M}(\xi) = \mathfrak{g}(\xi)\,\Omega^{\alpha\beta}_{\mathcal{G}}(\xi) \;-\; \chi^{\mathcal{G}}_{\mathrm{src}}(\xi)\,D_M\,\mathcal{G}^{\alpha\beta,M}(\xi) \;+\; \sigma^{\mathcal{G}}_{\mathrm{src}}(\xi)\,\mathcal{G}^{\alpha\beta}(\xi) \tag{26}$$

$$D_M\,\mathcal{R}^{\rho\omega,M}(\xi) = \mathfrak{r}(\xi)\,\Omega^{\rho\omega}_{\mathcal{R}}(\xi) \;-\; \chi^{\mathcal{R}}_{\mathrm{src}}(\xi)\,D_M\,\mathcal{R}^{\rho\omega,M}(\xi) \;+\; \sigma^{\mathcal{R}}_{\mathrm{src}}(\xi)\,\mathcal{R}^{\rho\omega}(\xi) \tag{27}$$

$$D_M\,\mathcal{V}^{\nu\kappa,M}(\xi) = \mathfrak{v}(\xi)\,\Omega^{\nu\kappa}_{\mathcal{V}}(\xi) \;-\; \chi^{\mathcal{V}}_{\mathrm{src}}(\xi)\,D_M\,\mathcal{V}^{\nu\kappa,M}(\xi) \;+\; \sigma^{\mathcal{V}}_{\mathrm{src}}(\xi)\,\mathcal{V}^{\nu\kappa}(\xi) \tag{28}$$

and the coupling equations (16)–(18) *with ∂_t replaced by the temporal component of D_M.*

Remark 7 (Interpretation)**.** *The promotion to gauge-covariant derivatives implements the Geometry paper's central result: the good will (the connection field $\mathcal{C}_M$) enforces frame-independence of ethical evaluation across locally varying evaluative vocabularies. The gauge-covariant Rawls-Santian equations ensure that the ethical field dynamics—not merely the field configurations—are objective in the sense of the objectivity-as-invariance principle. The coupling operators $\mathcal{C}^{\mathcal{A}\to\mathcal{B}}$ must themselves be covariant (commuting appropriately with D_M), which constrains their form and ensures that the inter-field coupling is a gauge-invariant feature of the ethical reality.*

The connection $\mathcal{C}_M$ takes values in the Lie algebra of the ethical gauge group, which—as argued in the main paper (§4.8.3)—has generators along all three ethical dimensions (axiological, deontic, aretaic) and is plausibly nonabelian. When the connection is nonabelian, the gauge-covariant derivative D_M introduces self-interaction terms (arising from the commutator $[\mathcal{C}_M, \cdot\,]$) that have no analog in the ordinary-derivative formulation. These self-interaction terms encode the objective cross-dimensional content that the main paper identifies as the "seal of objectivity" on the inter-field coupling: the part of the coupling that is certified as gauge-invariant by the connection's cross-dimensional curvature.

A.7 Structural Features Distinctive to the Ethical Case

The Rawls-Santian equations differ from Maxwell's equations in five structurally significant respects, each reflecting a genuine feature of the ethical domain identified in the companion papers:

1. **Three fields, not two.** Maxwell's equations couple two fields (**E** and **B**); the Rawls-Santian equations couple three ($\mathcal{G}$, $\mathcal{R}$, $\mathcal{V}$). This yields six coupling directions (each with its own coupling operator and effective permeability) rather than two, and the multiplicative coupling structure generates emergent three-way dynamics (virtuous and vicious spirals) with no two-field analog.

2. **Multiplicative coupling.** In Maxwell's equations, each curl equation involves a single other field additively. In the Rawls-Santian equations, each coupling equation involves the *product* of contributions from the other two fields. This multiplicative structure is what generates the non-additive dynamics: when all contributions are positive, the product amplifies faster than any sum; when one contribution collapses, the product propagates the collapse more aggressively.

3. **Damping and driving terms.** Maxwell's equations in vacuum have no dissipation or external driving. The Rawls-Santian equations include both, reflecting the agent-dependent character of the ethical field: without active maintenance (driving), the ethical fields dissipate (damping); with agential intervention, the fields can be sustained and amplified beyond what the medium's base permeability would support.

4. **General (asymmetric) tensors.** The electromagnetic field tensor $F_{\mu\nu}$ is antisymmetric ($F_{\mu\nu} = -F_{\nu\mu}$). The ethical field tensors $\mathcal{G}^{\alpha\beta}$, $\mathcal{R}^{\rho\omega}$, $\mathcal{V}^{\nu\kappa}$ are general: $\mathcal{F}^{ij} \neq \mathcal{F}^{ji}$ in general, reflecting the directional asymmetry of ethical coupling. This means the ethical fields carry more coupling degrees of freedom than the electromagnetic field (for an $n \times n$ tensor: n^2 independent components rather than $n(n-1)/2$).

5. **Field-valued permeabilities.** The electromagnetic permeabilities μ_0 and ε_0 are universal constants. The ethical permeabilities $\mathfrak{r}(\xi)$, $\mathfrak{g}(\xi)$, $\mathfrak{v}(\xi)$ are field-valued: they vary across the ethosystemic manifold, reflecting the situation-dependence of the strength of ethical field propagation. This is the formal expression of the claim that the ethical coupling is context-sensitive in a way that electromagnetic coupling is not.

Summary Table

Feature	**Maxwell-Heaviside (EM)**	**Rawls-Santian (Ethical)**
Coupled fields	2 ($\mathbf{E}$, $\mathbf{B}$)	3 ($\mathcal{G}$, $\mathcal{R}$, $\mathcal{V}$)
Field type	Antisymmetric rank-2	General rank-2
Coupling structure	Additive	Multiplicative
Coupling directions	2	6
Permeabilities	Universal constants	Field-valued functions
Damping/driving	Absent	Present
Propagation medium	Vacuum (fixed)	Ethosystem (dynamical)
Wave speed	$c = 1/\sqrt{\mu_0 \varepsilon_0}$	$c_{\text{eth}} = 1/\sqrt{\mathfrak{r}\,\mathfrak{g}\,\mathfrak{v}}$
Gauge structure	Abelian ($U(1)$)	Nonabelian

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Stephon Alexander's *The Jazz of Physics* (2016) inspired me (a) to pursue in earnest the project of formally describing ethical fields and their coupled dynamics by thinking of them as analogous to formally described physical ones, and (b) to explore disanalogies between physics and ethics as opportunities for innovation. Jordan Ellenberg's argument in *Shape* (2021) that new geometry serves as an independent locus of rational authority helped inspire the idea of crafting a formally specified ethical ideal (i.e., syntegrity) that aspires to objective, structural authority because it is inherently geometric: it specifies the condition under which all ethical dimensions jointly flourish due to their positive couplings. G.E. Moore's discussion of organic unities in *Principia Ethica* (1903) (wholes whose value is not equal to the sum of the value of their parts) inspired me to take non-additive ethical relations seriously. In this paper's formalization, the synergistic or antagonistic off-diagonal couplings of ethical tensor fields encode these non-additive relations.

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